Exercises

1. This problem explores the structure of metric spaces.

   (a) Given a metric space (X,d), a point $x \in X$, and a real number $r > 0$, show that the neighborhood $N(x,r)$ is an open set.

   (b) Let S_d be the set of signals that can appear in a one-sided discrete event system, where $T = [0,\infty)$. (S_d,d) is a metric space, where d is the Cantor Metric. Show that S_d is Hausdorff, meaning that given any $s_1, s_2 \in S_d$ such that $s_1 \neq s_2$, there are open sets U_1 and U_2 in S_d such that $s_1 \in U_1$, $s_2 \in U_2$, and $U_1 \cap U_2 = \emptyset$.

2. Consider the model shown in figure 6.2.

   (a) Prove that this system is not discrete.

   (b) Construct a model in the DE domain in Ptolemy II similar to the one in figure 6.2 with the following properties:

      i. it has a feedback loop where no actor is delta causal,
      ii. time diverges (it has events at times greater than any finite time), and
      iii. the system is discrete (there is no Zeno condition).

   This demonstrates that the condition requiring a delta-causal actor in a feedback loop is only sufficient, not necessary, to prevent Zeno conditions.

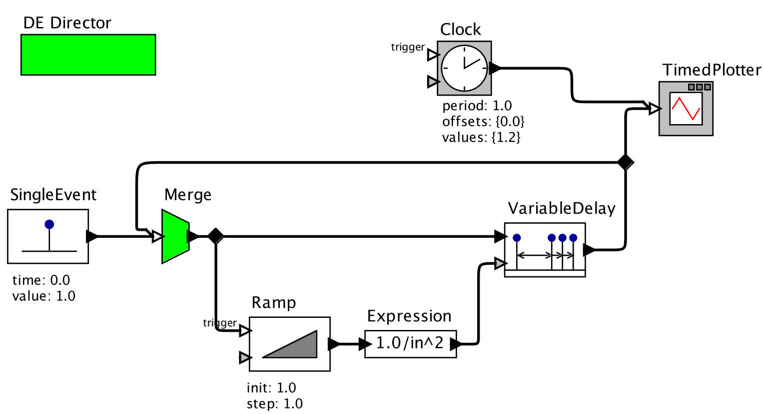

Figure 6.2: A discrete-event model that exhibits Zeno behavior.

3. This problem explores properties of the CPO semantics of DE. Suppose the tag set $T = \mathbb{R} \times \mathbb{N}$ is superdense time. Consider an actor A that delays each input event by one index. That is, given any input signal s, the non-absent events in the output signal are given by

$$\{((t, n+1), v) \mid ((t, n), v) \in s \text{ and } v \neq \varepsilon\}$$

   (a) Show that A is (Scott) continuous and discrete.

   (b) Show that A is not strictly causal under the CPO semantics.