Homework 4

EE 290n - Advanced Topics in Systems Theory

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1. For two posets A and B, a function $f: A \rightarrow B$ is an **order embedding** if $a \leq a' \iff f(a) \leq f(a')$. A function $f: A \rightarrow B$ is an **order isomorphism** if it is onto and an order embedding.

   Recall that a function $f: A \rightarrow B$ is **one-to-one** (or **injective**) if for all $a, a' \in A$,

   $$a \neq a' \Rightarrow f(a) \neq f(a').$$

   That is, no two distinct values in the domain yield distinct values in the codomain.

   Given a function $f: A \rightarrow B$ let $\hat{f}: \wp(A) \rightarrow \wp(B)$ denote the **image function**, defined by

   $$\forall A' \subset A, \quad \hat{f}(A') = \{b \in B \mid \exists a \in A' \text{ such that } f(a) = b\}.$$

   In words, $\hat{f}$ takes a set of arguments to f and returns the set of results. The returned result $\hat{f}(A')$ is called the **image** of A' under function f. The **range** of a function $f: A \rightarrow B$ is simply the image of its domain, $f(A)$.

   A function $f: A \rightarrow B$ is **onto** (or **surjective**) if $\hat{f}(A) = B$.

   (a) Show that if $f: A \rightarrow B$ is an order embedding, then f is one-to-one.

   (b) Show that if $f: A \rightarrow B$ is an order isomorphism, then there is an order isomorphism $g: B \rightarrow A$.

2. Consider the model shown in figure 1. This model is accessible at the following URL:

   `http://embedded.eecs.berkeley.edu/concurrency/lectures/Models11/zeno.xml`

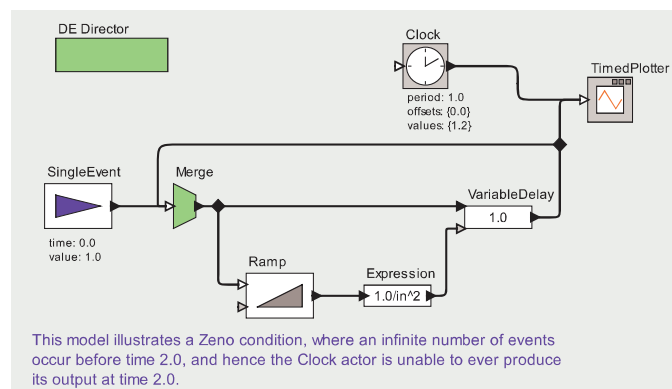   Prove that this system is not discrete.

Figure 1: A discrete-event model that exhibits Zeno behavior.

3. Construct a model in the DE domain in Ptolemy II similar to the one in figure 1 that has a feedback loop where no actor is delta causal, and yet the system is discrete (there is no Zeno condition).

4. Given a metric space (A, d), show that for all $a, b \in A$

$$d(a, b) \geq 0.$$

5. Prove that an ultrametric is a metric.