Proceedings of the 36th
Conference on Decision & Control
San Diego, California USA • December 1997

Cut-off in Engine Control: a Hybrid System Approach

A. Balluchi† M. Di Benedetto‡ C. Pinello†
C. Rossi§ A. Sangiovanni–Vincentelli†¶

†PARADES, Via San Pantaleo, 66, 00186 Roma, Italy, balluchi,pinello,alberto@parades.rm.cnr.it
‡Dipartimento di Ingegneria Elettrica, Università dell'Aquila, Poggio di Roio, 67040 L'Aquila, Italy, dibenede@giannutri.caspur.it
§Magneti Marelli S.p.A., Via del Timavo, 33, 40134 Bologna, Italy, Carlo.Rossi@bologna.marelli.it
¶Department of Electrical Engineering and Computer Science, University of California at Berkeley, CA 94720

Abstract

A novel approach to the control of an automotive engine in the cut-off region is presented. First, a hybrid model which describes the torque generation mechanism and the power-train dynamics is developed. Then, the cut-off control problem is formulated as a hybrid optimization problem, whose solution is obtained by relaxing it to the continuous domain and mapping its solution back into the hybrid domain. A formal analysis as well as simulation results demonstrate the properties and the quality of the control law.

1 Introduction

The overall goal of engine control in the automotive industry is to offer appropriate driving performance (e.g., speed, comfort, safety) while minimizing fuel consumption and emissions. Engine behavior is in general partitioned into regions of operation where appropriate control action are applied to yield the desired result [5]. The region of operation considered here is characterized by the driver who, by releasing the gas pedal, requests no torque to the engine. An obvious strategy to minimize gas consumption and emissions when no torque is requested is to shut fuel injection off, an operation called *cut-off*. Unfortunately, cutting off fuel injection as soon as the gas pedal is released, causes a sudden torque reduction that may result in unpleasant oscillations compromising driving comfort. A heuristic rule-based control strategy commonly used in industry consists of delaying throttle closure and, when air quantity is below a given threshold, reducing gradually fuel injection to zero, with an 'open loop' law that does not depend on the state of the system.

As is often the case, heuristic rule-based controls need extensive tuning, yield satisfactory solutions only in a limited range of operations and are hardly optimal with respect to the emissions and fuel consumption. In this paper, we introduce a novel, theoretically sound, closed-loop approach to cut-off control. The "plant" consists of two parts: the engine responsible for torque generation, modeled as a combination of an extended Finite State Machine and of a discrete-time system, and the power-train modeled as a fifth-order linear continuous-time system. The goal is to control the evolution of the system from an initial condition (the state of the system when the gas pedal is released) to cut-off minimizing the amplitude of the undesired oscillations. The control action is on fuel injection, the input to the engine, occurring only once per engine cycle for each piston. The torque generated is the output of the engine and the input to the power-train. The power-train output contains the potential oscillatory behavior to be minimized as well as the angular velocity of the crankshaft that determines the timing of the torque generation by the engine. Consequently, the control problem is a hybrid problem consisting of a discrete and a continuous system tightly linked. The control problem is further complicated by the delay between the time in which the decision on the quantity of fuel to be injected is taken (at the beginning of the exhaust phase) and the time the effect of this decision takes place (during the next expansion phase). Hybrid control albeit the subject of intensive study in the past few years (see for some interesting results [7], [10], [8], [6]) is still in its infancy. A well developed control design technique has not yet appeared. Our approach to the hybrid problem at hand is to relax the problem to the continuous domain assuming that torque generation can take continuous values. A sliding-mode control (see [9]) which solves the continuous control problem is devised. Then, the solution obtained in the continuous domain is mapped back into the discrete domain. The solution in the hybrid domain is demonstrated to yield a behavior that is close (within a precisely specified bound) to the behavior of the control in the continuous case. From simulations and preliminary analysis of our industrial partner, the solution achieved by this method appears to be far superior than the present, heuristic approach both in performance and emission control. The paper is organized as follows: In section 2, the torque-generation and power-train models are presented and the precise formulation of the optimal control problem is offered. In section 3 the optimal solution to the continuous control problem is presented. In section 4, a hybrid control scheme based on the continuous approximation is proposed to yield a solution demonstrably close to the upper bounding optimal continuous solution. Simulation results are reported in section 5. A comparison with the presently used control methods is offered with some concluding remarks and future work in section 6.

2 Problem formulation

2.1 Plant model

In this paper we deal with 4-stroke 4-cylinder gasoline engines with electronic injection and spark control [4], [3]. Consider the single cylinder hybrid model $\mathcal{M}$ in figure 1 consisting of: 1) a Finite State Machine (FSM) which describes, the evolution through the *4 phases (states)* of intake (I), compression (C), expansion (E) and exhaust (H); 2) a Discrete Event system (DE) modeling *torque generation*; a Continuous Time system (CT) modeling the *power-train*. Samplers and zero-order holders in the interconnections between the DE and CT models are implicitly included. The transitions of the FSMs occur when a piston reaches the bottom or top dead point. The guard condition enabling the transition is expressed in terms of the piston position $\tilde{\phi}$ measured on the crankshaft, considering the offset ϕ_{co} which corresponds to the angle the crank is mounted on the shaft.

The power-train model is a linear fifth order model, developed by Magneti Marelli Engine Control Division, a reduced order model of power-train dynamics describing the important phenomena involved in power-train oscillations of interest in cut-off control,

$$\left\{ \begin{array}{rcl} \dot{\zeta} & = & A^p \zeta + b^p u \\ \dot{\phi}_c & = & 6\,\omega_c \end{array} \right. \quad . \tag{1}$$

State $\zeta = [\alpha_b, \omega_p, \alpha_e, \omega_c]^T$ represents the engine block angle (radians), the wheel revolution speed (radians per second), the axle torsion angle (radians), the crankshaft revolution speed (rpm), and the crankshaft angle (degrees). The input signal u is the torque acting on the crank (Nm). Such signal is a complex function of time during the expansion phase that depends on the dynamics of the mix explosion. We assume for u a piece-wise constant model, set to zero during the I, C and H phases, and to an average value during the E phase.

Torque generation is modeled by the DE system, which is active at every FSM transitions. Let k be a counter of FSM transitions occurring at time t_k. The DE inputs are: the mass of air-fuel mix $q \in \mathbb{R}^+$ loaded in the intake phase (kg); a binary signal $j \in \{0, 1\}$ which indicates whether or not the fuel is present in the mix;

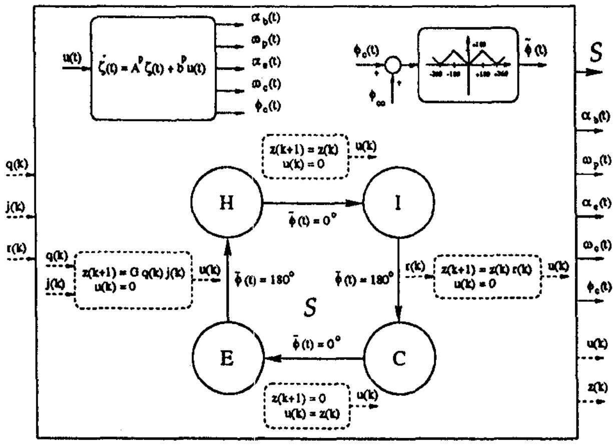

Figure 1: Hybrid model for a single cylinder.

the modulation factor $r \in [r_{min}, 1]$ due to not optimal spark timing[1]. The DE system output is the torque u applied to the crank. At the FSM transition $E \to H$ the DED reads its inputs q and j, and stores in its state $z \in \mathbb{R}$ the maximum amount of torque (Nm) achievable during the next E phase, obtained by the mix-to-torque gain G. Such value is corrected at the $I \to C$ transition by the modulation factor r due to the chosen spark advance. The DE output u is always zero except at the $C \to E$ transition when is set to the value stored in z.

In this paper we focus on the most relevant case of a 4 cylinder engine. Its model, referred to in the rest of the paper as $\mathcal{M}_{4cyl}$, is the composition of the cylinders' FSMs along with their torque generation DE model, while the power-train CT model is shared among all cylinders. Input signals $\mathbf{j} = [j_1, j_2, j_3, j_4]^T$ and $\mathbf{r} = [r_1, r_2, r_3, r_4]^T$ are properly synchronized with the corresponding DE models; we denote by $\mathcal{J}_d$ and $\mathcal{R}_d$ the classes of functions $\mathbb{N} \to \{0, 1\}^4$ and $\mathbb{N} \to [r_{min}, 1]^4$, feasible for $\mathbf{j}$ and $\mathbf{r}$. Signal q is instead shared among the cylinders. Crankshaft offsets ϕ_{co_i} are zeros. Model parameters for a commercially available four cylinder car are: $r_{min} = .6$, $G = 2.5\,10^4$, $b^p = [1.52\,10^{-3}, 0, -1.33\,10^{-3}, 47.8]^T$, and

$$A^p = \begin{bmatrix} -34.7 & -6.34\,10^{-2} & 12.6 & 8.11\,10^{-4} \\ 19.2 & -.611 & 412 & 7.77\,10^{-3} \\ 30.3 & -.944 & -11.0 & 1.23\,10^{-2} \\ -8290 & 262 & -52.2\,10^{3} & -3.84 \end{bmatrix}$$

with eigenvalues $\lambda \pm j\mu = -3.32 \pm j25.7$, $\lambda_1 = -0.0757$ and $\lambda_2 = -43.5$.

[1] Since combustion is not instantaneous, spark ignition is given before the piston concludes the compression phase, corresponding to a specified spark advance. For a given mix, there exists an optimal spark advance which produces the maximum torque.

2.2 The optimization problem

The optimization problem is to control fuel injection and spark advance so that vehicle acceleration peaks are minimized during the cut-off operation, for a given decreasing sequence $q_a(k)$ of air intakes. Assuming vehicle speed equal to wheel speed, vehicle acceleration is $a(t) = R\,\dot{\omega}_p(t)$, where R is the wheel radius. To isolate oscillations from monotone behavior, the following state transformation is applied: set $[x, x']^T = T\zeta$ given by the eigenvectors of A^p, and rewrite (1) as

$$\begin{bmatrix} \dot{x} \\ \dot{x}' \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix} + \begin{bmatrix} b \\ b' \end{bmatrix} u \qquad (2)$$

where $A = \begin{bmatrix} \lambda & -\mu \\ \mu & \lambda \end{bmatrix}$, $b = [26.4, -35.0]^T$ and $D = \mathrm{diag}(\lambda_1, \lambda_2)$. Denoting by $c \in \mathbb{R}^{1\times 2}$ the product between the second row of A^p and the first two columns of T^{-1} times R, the oscillating component $\tilde{\dot{\omega}}_p(t)$ of $\dot{\omega}_p(t)$ is given by $\tilde{\dot{\omega}}_p(t) = c\,x(t)$. Hence, the acceleration oscillation can be expressed as

$$\tilde{a}(t) = c\,x(t) \qquad (3)$$

with $c = [-2.46\,10^{-4}, 1.33\,10^{-2}]$. The optimization problem can be formulated in the x state space as follows

Problem P.1:

$$\min_{\substack{\mathbf{j} \in \mathcal{J}_d \\ \mathbf{r} \in \mathcal{R}_d}} \|\tilde{a}\|_\infty = \min_{\substack{\mathbf{j} \in \mathcal{J}_d \\ \mathbf{r} \in \mathcal{R}_d}} \sup_{t \ge 0} |\tilde{a}(t)| \qquad (4)$$

$$\text{subject to:} \begin{cases} \text{Dynamics of Hybrid Model } \mathcal{M}_{4cyl} \\ x(0) = x_0, \ \lim_{t\to\infty} x(t) = [0,0]^T \\ q(k) = q_a(k) \end{cases}$$

3 Continuous-time model solution

The main difficulties solving Problem P.1 are that the plant to be controlled is hybrid and that the input signals are bounded. In [7], [1], some foundations of hybrid control theory are laid out and some examples of hybrid control problems described but no generic solution s given that can be directly applied to our problem. Our simple strategy is to first address a relaxed problem, considering only the CT problem and, then, to map the solution back to the hybrid domain. The relaxed problem in the CT domain is as follows:

Problem P.2:

$$\min_{u \in \mathcal{U}} \|\tilde{a}\|_\infty = \min_{u \in \mathcal{U}} \sup_{t \ge 0} |c\,x(t)| \qquad (5)$$

$$\text{subject to:} \begin{cases} \dot{x}(t) = Ax(t) + bu(t) \\ x(0) = x_0, \ \lim_{t\to\infty} x(t) = [0,0]^T \end{cases} \qquad (6)$$

where feasible input signals $u(t)$ belong to the class:

$$\mathcal{U} = \{u : [0, +\infty) \to \mathbb{R} \mid 0 \le u(t) \le M, \ \forall t \ge 0\} \qquad (7)$$

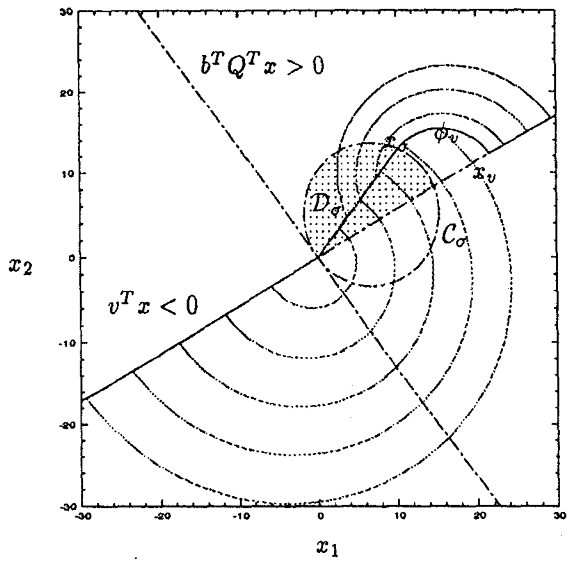

Figure 2: Trajectories under the proposed VS control for the torque generation continuous model.

where $M = Gq_a^o$ and q_a^o is the steady-state air quantity after pedal release.

3.1 Optimal reaching strategy

Problem P.2 involves the selection of a function u so that the functional $\|\tilde{a}\|_\infty$ is minimized. In this section, we present a technique that constructs the function u point by point. Let us introduce the optimization problem in $\mathbb{R}$:

Problem P.2aux: Given $t' > 0$,

$$\min_{u(t') \in [0,M]} \frac{\dot{x}^T(t')}{\|\dot{x}(t')\|} \frac{x(t')}{\|x(t')\|} \qquad (8)$$

where the cost function is the misalignment between the vectors in $\mathbb{R}^2$ $\dot{x}(t')$ and $x(t')$. To avoid overly complex notation, from here on, we will consider u and $u(t')$, $\dot{x}$ and $\dot{x}(t')$, x and $x(t')$ identical. The minimum of the cost function is -1 and it is achieved if there exists a feasible control $u \in [0, M]$ such that $\dot{x}$ points towards the origin. Let $Q \in \mathbb{R}^2$ be the $+\pi/2$ rotation matrix and consider the vector $x_\perp = Qx$. Vector $\dot{x}$ is colinear to x if and only if $x_\perp^T(Ax + bu) = 0$, that is if

$$u = -\frac{x_\perp^T A x}{x_\perp^T b} = -\frac{x^T Q^T A x}{x^T Q^T b}. \qquad (9)$$

This condition is clearly necessary but not sufficient for u to be a solution. First we have to check whether $\dot{x}$ points towards the origin or in the opposite direction and then whether control signal u as in (9) is feasible, i.e., $u \in [0, M]$. Under control (9), $\dot{x}$ points away from the origin $\forall x \in \mathcal{A}_v = \{x \in \mathbb{R}^2 \mid (v^T x)(b^T Q^T x) > 0\}$, with

$$v = QA^{-1}b\,\|QA^{-1}b\|^{-1}. \qquad (10)$$

$\mathcal{A}_v$ is bounded by the line passing through the equilibrium points ($v^T x = 0$) and the line collinear to b, see figure 2. Control signal u in (9) is feasible if x belongs to the circle[2]

$$\mathcal{C}_\sigma \;=\; \left\{x \in \mathbb{R}^2 \,|\;\; x^T Q^T (Ax + bM) \leq 0\right\}. \qquad (11)$$

Hence control (9) is optimal for $x \in \mathcal{D}_\sigma = \mathcal{C}_\sigma / \mathcal{A}_v$. Outside $\mathcal{D}_\sigma$, the optimal solution is a bang–bang control, i.e. $u \in \{0, M\}$. The two controls are equivalent when the corresponding velocities are aligned, i.e.

$$\exists\, \alpha \in \mathbb{R} \mid Ax \;+\; bM = \alpha Ax \quad \Longleftrightarrow \quad v^T x = 0.$$

An optimal solution for problem (8) is then the control law

$$\bar{u} = \begin{cases} -\frac{x^T Q^T A x}{x^T Q^T b} & \text{if } \; x \in \mathcal{D}_\sigma \\ \begin{cases} 0 & \text{if } \; v^T x > 0 \\ M & \text{if } \; v^T x \leq 0 \end{cases} & \text{if } \; x \notin \mathcal{D}_\sigma \end{cases} . \qquad (12)$$

The solutions to Problem P.2aux can be used to yield the optimal solution to P.2.

Proposition 1 *Given an initial condition $x(0) = x_0$, the trajectory η^0 from x_0 to the origin generated by the law $u \in \mathcal{U}$ as in (12), optimal for Problem P.2aux, is also optimal for problem P.2.*

3.2 Sliding–mode with bounded control signals

Even if it is theoretically optimal, control law (12), may result in an unsatisfactory closed loop behavior since, inside $\mathcal{D}_\sigma$, may yield slow convergence and have robustness problems depending on the initial states. Moreover, since the available control are $j \in \{0, 1\}$ and $r \in [r_{min}, 1]$, (12) may not be feasible. Applying a classical sliding mode approach (see [9]) a sub-optimal, but robust, solution for problem P.2 is derived. Consider system (6) and introduce the linear manifold

$$\sigma(x) \;=\; \gamma^T x \;=\; 0\,. \qquad (13)$$

Given a point x on the manifold (13), there exists a variable structure control in $\mathcal{U}$ which satisfies the sliding–mode existence condition $\sigma(x)\dot{\sigma}(x) < 0$ iff

$$\min_{u \in [0,M]} \left\{\gamma^T (Ax + bu)\right\} \;\; \max_{u \in [0,M]} \left\{\gamma^T (Ax + bu)\right\} < 0.$$

The locus of points on (13) where a sliding regime can be achieved is then given by the open segment

$$\mathcal{S}_\sigma = \{x \in \mathbb{R}^2 \,|\, \sigma(x) = 0 \wedge (\gamma^T Ax)\gamma^T (Ax + bM) < 0\} \qquad (14)$$

When vector γ rotates in the plane, segment $\mathcal{S}_\sigma$ describes the circle $\mathcal{C}_\sigma$ in (11). The extreme points of $\mathcal{S}_\sigma$ are the origin and the point x_σ given by the intersection of $\mathcal{C}_\sigma$ and $\sigma(x) = 0$, see figure 2. The dynamics during the sliding regime can be found by means of the well-known method of equivalent control: $\dot{x} = [I - b(\gamma^T b)^{-1} \gamma^T] Ax$ for $\sigma(x) = 0$. Given a desired eigen-value λ_c, the sliding manifold is then obtained as

$$\gamma \;=\; \frac{N^T}{\|[-\lambda_c \;\; 1]N\|} \begin{bmatrix} -\lambda_c \\ 1 \end{bmatrix} \qquad (15)$$

where N is the state transformation which leads to the canonical controllable form ([2]). Note that, if $\gamma = v$ then $\lambda_c = 0$. Hence, given any sliding segment inside $\mathcal{C}_\sigma$, the closed loop dynamics is stable if and only if it belongs to $\mathcal{D}_\sigma$. Assume $\gamma^T b > 0$. Inside $\mathcal{D}_\sigma$, control $u = 0$ ($u = M$) provides attractiveness of $\mathcal{S}_\sigma$ from all initial points $x_0 \in \mathcal{D}_\sigma$ with $\sigma(x_0) > 0$ ($\sigma(x_0) < 0$, resp.). Finally, to properly define the closed loop control law, the switching curve $v^T x = 0$ established in (12) (outside $\mathcal{D}_\sigma$) has to be linked with the sliding manifold (13) (inside $\mathcal{D}_\sigma$). Consider the arc of forced evolution φ_v of system (6) under the control $u = M$, arriving at x_σ and leaving from the first intersection point, x_v, with line $v^T x = 0$ (see figure 2)

$$\varphi_v \,:\; x(\alpha) = e^{A\alpha} x_\sigma - \left(I - e^{A\alpha}\right) A^{-1} bM \quad \alpha \in [-\tau_v, 0]\,. \qquad (16)$$

A suitable choice for the switching function is given by

$$\sigma(x) = \begin{cases} (\gamma^T v)\, v^T x & \text{if } v_\perp^T x \leq 0 \\ \gamma^T x & \text{if } 0 < v_\perp^T x \leq v_\perp^T x_\sigma \\ \gamma^T (x - \tilde{x}), \text{ with} & \\ \begin{cases} v_\perp^T (\tilde{x} - x) = 0 \\ \tilde{x} \in \varphi_v \end{cases} & \text{if } v_\perp^T x_\sigma < v_\perp^T x \leq v_\perp^T x_v \\ (\gamma^T v)\, v^T x & \text{if } v_\perp^T x_v < v_\perp^T x \end{cases} \qquad (17)$$

where $v_\perp = Qv$ and $\tilde{x}$ is the projection of x on φ_v parallel to $v_\perp^T x = 0$. The resulting VS control is then

$$u_{vsc} = \begin{cases} 0 & \text{if } \; \sigma(x) > 0 \\ M & \text{if } \; \sigma(x) < 0 \end{cases} . \qquad (18)$$

The trajectories generated by the VS control (18) along with the sliding manifold are shown in figure 2.

4 Hybrid system control scheme

The continuous approach described in section 3, cannot be directly applied to the hybrid model $\mathcal{M}_{4cyl}$ introduced in section 2, due to the fact that feasible control actions for the CT system in $\mathcal{M}_{4cyl}$ are piece-wise constant and synchronized with the crank angle according to the DE system. This constraint acts as a nonlinearity in the feedback loop. In classical VS theory it is shown that non-linearities produce a real sliding motion with a finite frequency chattering phenomenon around the sliding manifold.

Let us assume that air dynamic is negligible so that in problem **P.1** the air quantity q_a can be approximated by the value with pedal released q_a^o. Let then

[2] $\mathcal{C}_\sigma$ has center in $-\frac{M}{2\mu} Q^T b$ and radius $\frac{M}{2\mu}\|b\|$. Its boundary contains the origin (with tangent collinear to vector b) and $x_e = -A^{-1} bM$, the equilibrium point with $u = M$.

$M = Gq_a^o$. The continuous solution is now mapped into a feasible control law for model $\mathcal{M}_{4cyl}$ in terms of signal $j_i(k)$ and $r_i(k)$. According to (18), $r_i(k)$ is set to 1 to realize the bang-bang control. Due to the delay of torque generation represented by the DE model, control signal $j_i(k)$, at current time t_k corresponding to a $E \to H$ transition of the i–th cylinder, will produce, at the next expansion run of the i–th cylinder, the torque $u(k+3)$. Such signal will feed the CT system during the interval $[t_{k+3}, t_{k+4})$, steering the CT state from $x(t_{k+3})$ to $x(t_{k+4})$. Assuming $x(t)$ available, an estimate $\hat{x}(k+3)$ of the point the x state will reach at time $t = t_{k+3}$ is computed at the each instant t_k, by forward integration of the CT model from $x(t_k)$ on the time intervals $[t_k, t_{k+1})$, $[t_{k+1}, t_{k+2})$ and $[t_{k+2}, t_{k+3})$ knowing the controls $j_l(k-3)$, $j_m(k-2)$ and $j_n(k-1)$. Hence, evaluating, by means of a further integration, the estimates $\hat{x}_0(k+4)$ and $\hat{x}_M(k+4)$ of the two points reachable by the state x at time t_{k+4} under control $j_i(k) = 0$ and $j_i(k) = 1$ respectively, the hybrid control is chosen as

$$j_i(k) = \begin{cases} 0, \text{ if } |\sigma(\hat{x}_M(k+4))| \geq |\sigma(\hat{x}_0(k+4))| \\ 1, \text{ if } |\sigma(\hat{x}_M(k+4))| < |\sigma(\hat{x}_0(k+4))| \end{cases} \tag{19}$$

The sliding motion produced by control law (19) on the hybrid system $\mathcal{M}_{4cyl}$ is characterized by the following proposition.

Proposition 2 *Let*

$$\begin{aligned} &\rho = \frac{[(\gamma_1^T e^{-A\tau} v)^2 + (\gamma^T v)^2]^{\frac{1}{2}}}{|2\gamma^T v|} \gamma^T \left(I - e^{A\tau}\right) x_e, \\ &\text{with } x_e = -A^{-1} bM \text{ and } \tau = \frac{30}{\omega_c(0)} \\ &\text{and } B_\rho = \{x| \parallel x \parallel \leq \rho\}. \end{aligned} \tag{20}$$

There exists ω_c^{min}, such that, for initial crank velocity $\omega_c(0) > \omega_c^{min}$ and under the hypothesis that $\omega_c(t) = \omega_c(0)$ for all t such that $x(t) \notin B_\rho$, the variable structure control law (19) yields in finite time a sliding motion on the segment of the switching manifold (13) which lies on $\gamma^T x = 0$. The sliding motion is bounded within a layer of amplitude

$$\delta = -\frac{1}{2}\gamma^T \left(I - e^{A\tau}\right) x_e \tag{21}$$

of the manifold. In finite time, the trajectory enters B_ρ, inside which the state converges asymptotically to the origin in free evolution.

5 Simulation results

In addition to the theoretical results provided in the previous sections, a number of experiments have been performed to show the actual behavior of the control law on the model under investigation. The simulation exercise itself is non trivial since tools for the simulation of hybrid systems of a certain complexity are not readily available. Dr. Alberto Ferrari in our research group has developed a general purpose prototype hybrid simulator (to be described in a forthcoming publication). The simulator is based on the combination of Ptolemy, a design tools for the simulation of heterogeneous systems developed at the Department of EECS, University of California at Berkeley, with Xmath, a commercially available system by Integrated Sytems Inc. that is widely used in the automotive industry. The simulation is run on a model of a four cylinder combustion engine of a commercially available car. In figure 3 the simulated evolution of the CT system reduced state x is shown for a cut-off operation under the proposed control from the initial state $[\alpha_b(0), \omega_p(0), \alpha_e(0), \omega_c(0), \phi_c(0)]^T = [2.8\,10^{-3}, 60.7, 6.83\,10^{-3}, 4.67\,10^3, 0]^T$. In the x space such initial state corresponds to an equilibrium point for a given constant air quantity. At $t = 0$, due to pedal release, the air quantity drops to q_a^o. The injection control (19) steers then the state towards the sliding segment and produces the sliding motion convergent to the origin. Figure 4 shows the evolution of the real sliding motion around the sliding segment $\gamma^T x = 0$. The simulation confirms the theoretical results of Proposition 2: chattering is bounded within a layer of amplitude $\delta = 1.37$ given by (21), and the injection signal is disabled once the state enters the ball of radius $\rho = 1.58$ given by (20). The injection signal profile generated by the control law (19) is reported in figure 5. The oscillation reduction achieved by the proposed control is shown in figure 6 where the acceleration resulting from the proposed approach is compared to the behavior of immediate cut-off with no control.

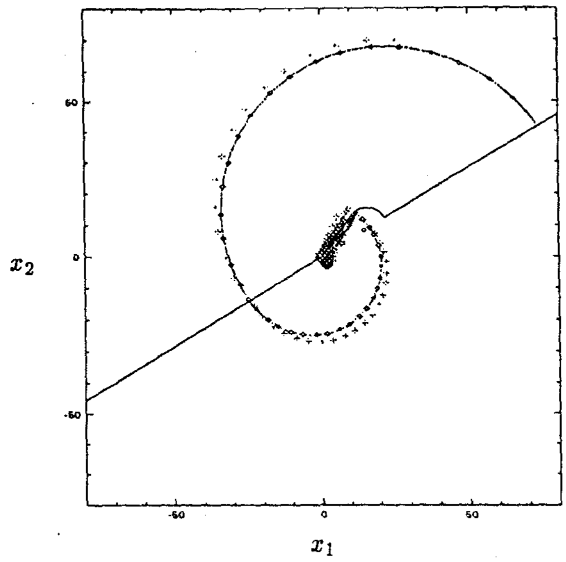

Figure 3: Evolution of the CTD in the x space during a cut-off operation under the proposed control.

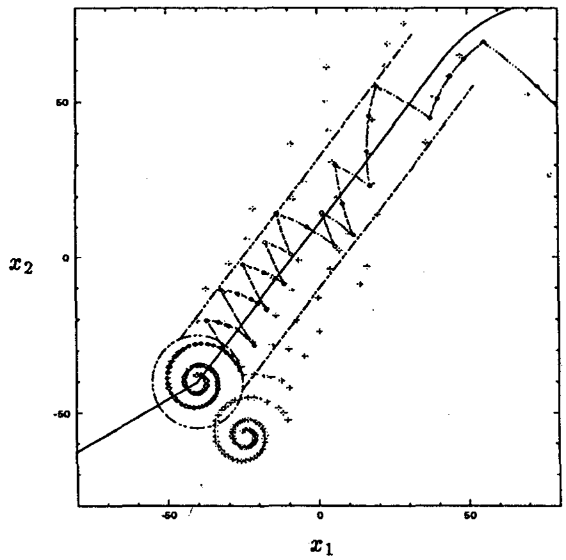

Figure 4: Chattering motion around the sliding segment $\gamma^T x = 0$. Trajectories are compared with the layer of amplitude δ and the sphere B_ρ.

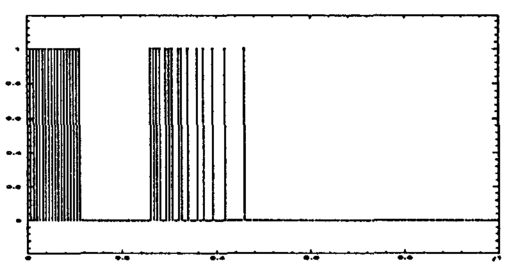

Figure 5: Injection signal profile.

6 Conclusions and future work

In this paper, we presented a novel approach to engine control in the cut-off region, based on a hybrid model of the torque generation and of the power-train dynamics in a four-stroke engine. A control problem on this hybrid system is defined and solved using a sequence of approximations. The properties of the control law so obtained have been rigorously characterized, thus offering better confidence on the quality of the results with respect to commonly used heuristic, open loop, approaches. In addition, since the control law is closed loop, expensive tuning processes can be avoided yielding a commercially appealing solution. Simulation shows effective oscillation control on a realistic case. We expect to run the first experiments on cars by the end of 1997 and to see the final version of the control laws in products shortly after. In addition, we expect to extend the approach to the problem of idle speed control that shares several key features with the cut-off control problem.

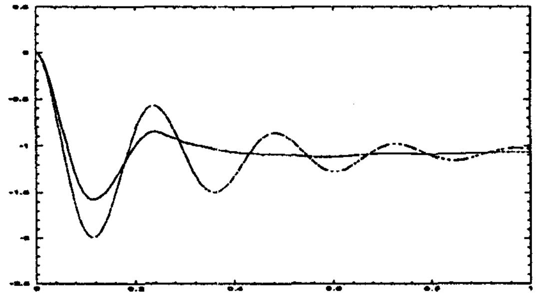

Figure 6: Acceleration profile for a controlled cut-off compared to an uncontrolled one.

7 Acknowledgments

This research has been partially sponsored by PARADES, a Cadence, Magneti-Marelli and SGS-Thomson GEIE, and by CNR. We wish to acknowledge the support of the management of Magneti Marelli, and, in particular of Drs. Pecchini, Mortara and Romeo, without which this work would not have been possible. Dr. Alberto Ferrari provided the simulation environment and the actual simulation results. Dr. Luca Benvenuti offered insightful comments on the results of the paper.

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