Homework 3

EE 290n - Advanced Topics in Systems Theory

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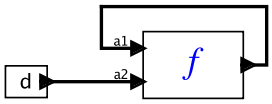
1. This problem studies a determinate PN actor with behavior similar in some ways to the non-determinate merge. Suppose D is a set of data values and $f\colon (D^{**})^2 \to D^{**}$ is a function defined by

$$f(s_1, s_2) = \underbrace{(e, e, \cdots, e)}_{N \text{ times}}$$

for all $s_1, s_2 \in D^{**}$, for some $e \in D$, and where $N = |s_1| + |s_2|$, the sum of the lengths of the sequences s_1 and s_2. If either input sequence is infinite, then the length of the output is infinite.

   (a) Suppose this function is used in the following PN model:



   where $d \in D^{**}$ is a finite sequence of length 5 produced by the left actor. What is the output of f?

   (b) Is this function monotonic? Continuous?

   (c) Is this function sequential?

Solution.

(a) The only fixed point is where the output is the infinite sequence $(e, e, \cdots)$.

(b) This function is monotonic and continuous.

(c) This function is not sequential.

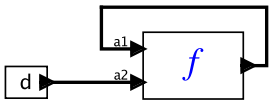
□

2. Suppose D is a set of data values and $A = \{\bot, \varepsilon\} \cup D$ is a flat CPO with $\bot$ at the bottom. Consider a function $f\colon A^2 \to A$ given by, for all $a_1, a_2 \in A$,

$$f(a_1, a_2) = \begin{cases} \bot & \text{if } a_1 = a_2 = \bot \\ a_1 & \text{if } a_1 \in D \\ a_2 & \text{otherwise} \end{cases}$$

   (a) Is this function strict?

   (b) Suppose this function is used in the following SR model:



where $d \in D$ is produced on every tick by the left actor. What is the output of f on each tick? What if d is replaced with ε?

(c) Show that this function is not monotonic.

Solution.

(a) The function is not strict. Even without knowing anything about the second argument, if it knows the first, it can assert the output.

(b) The only fixed point is where the output of f is d, because $f(d,d) = d$. If d is replaced with ε, then the only fixed point is ε.

(c) Note that for any $d_1, d_2 \in D$, $f(\bot, d_2) = d_2$ but $f(d_1, d_2) = d_1$. Thus, although $(\bot, d_2) \leq (d_1, d_2)$, we get that $f(\bot, d_2) \not\leq f(d_1, d_2)$. QED

□

3. Suppose D is a set of data values and $A = \{\bot, \varepsilon\} \cup D$ is a flat CPO with $\bot$ at the bottom. Consider a function $g\colon A^2 \to A$ given by, for all $a_1, a_2 \in A$,

$$g(a_1, a_2) = \begin{cases} \bot & \text{if } a_1 = a_2 = \bot \\ d & \text{if } a_1 \in D \text{ or } a_2 \in D \\ \varepsilon & \text{otherwise} \end{cases}$$

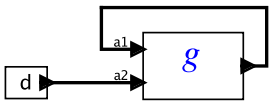
for some $d \in D$.

(a) Is this function strict?

(b) Is this monotonic? continuous?

(c) Is this function sequential?

(d) Suppose this function is used in the following SR model:



where $d \in D$ is produced on every tick by the left actor. What is the output of f on each tick? What if d is replaced with ε?

Solution.

(a) The function is not strict. It can assert a known output if one input is known to be in D.

(b) The function is not monotonic. Note that $(\perp, \varepsilon) \leq (d, \varepsilon)$ but $f(\perp, \varepsilon) = \varepsilon \not\leq f(d, \varepsilon) = d$.

(c) The function is not sequential. In particular, note that $f'(\perp, \perp) = \perp$. If it were sequential, then then would need to be true that either $(f'(d_1, \perp) = \perp$ or $(f'(\perp, d_2) = \perp$. That is, it would need to be true that the function was blocked on one of the two inputs. But neither of these is true for $d_1, d_2 \in D$.

(d) The output is d. If d is replaced by ε, then the output is ε.

□

4. This problem explores construction of various sequential control logic systems using the Ptolemy II SR director and basic SR actors. The basic SR actors are Default, When, Pre, NonstrictDelay, and EnabledComposite. You may also use other basic actors not specific to SR, such as Const, Sequence, Ramp, and any arithmetic or logic functions.

   (a) Use Sequence and When to construct an SR model that generates a sequence of values *true* interspersed with *absent* (ε). For example, produce the sequence

   $$(true, \varepsilon, \varepsilon, true, \varepsilon, true, true, true, \varepsilon) \ .$$

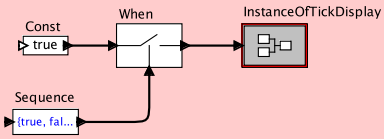
   (b) Create a composite actor IsAbsent that given any input sequence, produces an output *true* at every tick when the input is ε, and otherwise produces the output ε.

   (c) Create a model that can recognize the difference between single and double mouse clicks. That is, given a sequence like those generated by your model in part (a), produce an output *true* on an output port named *singleClick* if there are N or more ε following an input *true* on an input port named *click*. Otherwise, upon occurrence of a second *true* within N ticks, it outputs a *true* on a port named *doubleClick*. How does your model behave if given three values *true* within N ticks on input port *click*? To be concrete, you may implement this for the fixed $N = 3$, although a better design would have this as a parameter.

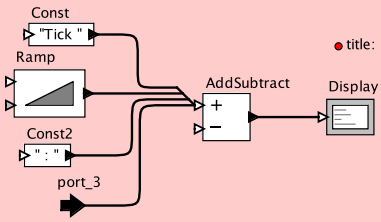
   (d) **Extra credit**: Redo (a)-(c) by writing a custom a Java actor for each of the three functions above. How does this design compare with the design implemented using primitive SR actors? Is it more or less understandable? complex?

Solution.

(a) The following model will do the job:



where the InstanceOfTickDisplay is the composite actor shown below, created to format the output:



The output is:

```
Tick 0 : true
Tick 1 :
Tick 2 :
Tick 3 : true
Tick 4 :
Tick 5 : true
Tick 6 : true
Tick 7 : true
Tick 8 :
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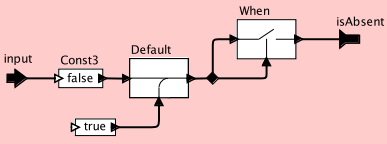
(b) The following model will do the job:



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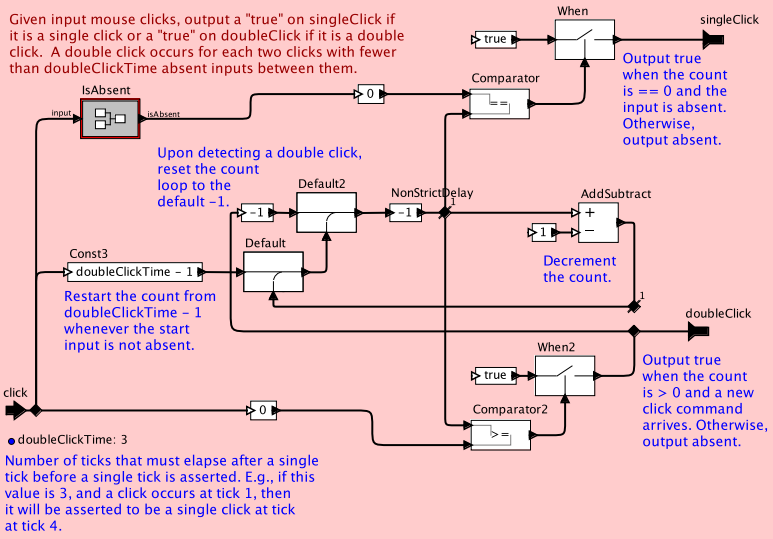
Solution.

(c) The following model will do the job:



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