**Homework 6**

EE 290n - Advanced Topics in Systems Theory

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1. For two posets $A$ and $B$, a function $f \colon A \to B$ is an **order embedding** if $a \leq a' \iff f(a) \leq f(a')$. A function $f \colon A \to B$ is an **order isomorphism** if it is onto and an order embedding. Two posets are **order isomorphic** if there is a order isomorphism from one to the other.

   (a) Show that if $f \colon A \to B$ is an order embedding, then $f$ is one-to-one.

   (b) Show that if $f \colon A \to B$ is an order isomorphism, then there is an order isomorphism $g \colon B \to A$.

2. Consider the model shown in figure 1. This model is accessible at the following URL:

   ```
   http://embedded.eecs.berkeley.edu/concurrency/2009/homework/HW6/Zeno.xml
   ```

   (a) Prove that this system is not discrete.

   (b) Construct a model in the DE domain in Ptolemy II similar to the one in figure 1 with the following properties:

       i. it has a feedback loop where no actor is delta causal,
       ii. time diverges (it has events at times greater than any finite time), and
       iii. the system is discrete (there is no Zeno condition).

   This demonstrates that the condition requiring a delta-causal actor in a feedback loop is only sufficient, not necessary, to prevent Zeno conditions.

3. Given a metric space $(A, d)$, show that for all $a, b \in A$ $d(a, b) \geq 0$.
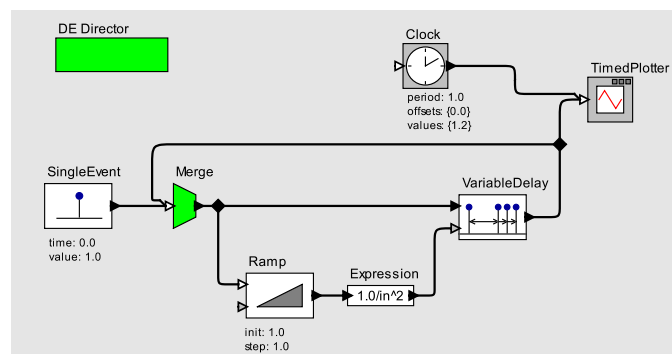
4. Prove that an ultrametric is a metric.



Figure 1: A discrete-event model that exhibits Zeno behavior.