Homework 5
EE 290n - Advanced Topics in Systems Theory
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1. Suppose $V$ is some set and $S = V^{**}$ is the set of finite and infinite sequences of elements of $V$. This exercise explores some of the properties of the CPO $S^n$ with the pointwise prefix order, for some non-negative integer $n$.

(a) Show that any two elements $a, b \in S^n$ that have an upper bound have a least upper bound.

(b) Let $U \subset S^n$ be such that no two distinct elements of $U$ are joinable. Prove that for all $s \in S^n$ there is at most one $u \in U$ such that $u \sqsubseteq s$.

(c) Given $s \in S^n$, suppose that $Q(s) \subset S^n$ is a joinable set where for all $q \in Q(s), q \sqsubseteq s$. Then show that there is an $s'$ such that $s = (\bigvee Q(s)).s'$.

2. Consider the model shown in figure 1. Assume that data types are all $V = \{0, 1\}$. Assume $f$ is a dataflow that implements an identity function and that Const is an actor that produces an infinite sequence $(0, 0, 0, \cdots)$. Obviously, the overall output of this model should be this same infinite sequence. The box labeled $g$ indicates a composite actor. Find firing rules and firing function $g$ for the composite actor to satisfy conditions 1 and 3 covered in class. Note that the composite actor has one input and two outputs.

3. Extra credit. In theory, dataflow models with only boolean data types, switch, select, and logic functions are Turing complete. A simple function that should be implementable, but is not easy to implement using such primitives, is one that, given a sequence $(v_1, v_2, \cdots)$ produces a sequence where every block of five inputs is reversed, yielding

$$(v_5, v_4, v_3, v_2, v_1, v_{10}, v_9, \cdots).$$

I am looking for elegant dataflow models using the dynamic dataflow (DDF) director in Ptolemy II (under ExperimentalDirectors). An extension of this would use integer data types.

![Figure 1: A model.](image-url)
and given three sequences $v = (v_1, v_2, \cdot)$, $(n_1, n_2, \cdot)$, and $(m_1, m_2, \cdot)$ that would behave as follows: for every integer $i > 0$, it would consume $n_i$ tokens from $v$ and push them onto a stack, then pop $m_i$ tokens from the stack (reversing their order) and produce them on the output. I am looking for an elegant dataflow model that performs this function. Note that I do not have a solution to this problem.