Homework 4
EE 290n - Advanced Topics in Systems Theory
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1. For two posets $A$ and $B$, a function $f: A \rightarrow B$ is an order embedding if $a \leq a' \iff f(a) \leq f(a')$. A function $f: A \rightarrow B$ is an order isomorphism if it is onto and an order embedding. Recall that a function $f: A \rightarrow B$ is one-to-one (or injective) if for all $a, a' \in A$,

$$a \neq a' \implies f(a) \neq f(a').$$

That is, no two distinct values in the domain yield distinct values in the codomain.

Given a function $f: A \rightarrow B$ let $\hat{f}: \wp(A) \rightarrow \wp(B)$ denote the image function, defined by

$$\forall A' \subset A, \quad \hat{f}(A') = \{ b \in B \mid \exists a \in A' \text{ such that } f(a) = b \}.$$ 

In words, $\hat{f}$ takes a set of arguments to $f$ and returns the set of results. The returned result $\hat{f}(A')$ is called the image of $A'$ under function $f$. The range of a function $f: A \rightarrow B$ is simply the image of its domain, $f(A)$.

A function $f: A \rightarrow B$ is onto (or surjective) if $\hat{f}(A) = B$.

(a) Show that if $f: A \rightarrow B$ is an order embedding, then $f$ is one-to-one.

(b) Show that if $f: A \rightarrow B$ is an order isomorphism, then there is an order isomorphism $g: B \rightarrow A$.

2. Consider the model shown in figure 1. This model is accessible at the following URL:

http://embedded.eecs.berkeley.edu/concurrency/lectures/Models11/zeno.xml

Prove that this system is not discrete.

![Diagram](http://embedded.eecs.berkeley.edu/concurrency/lectures/Models11/zeno.xml)

Figure 1: A discrete-event model that exhibits Zeno behavior.
3. Construct a model in the DE domain in Ptolemy II similar to the one in figure 1 that has a feedback loop where no actor is delta causal, and yet the system is discrete (there is no Zeno condition).

4. Given a metric space \((A, d)\), show that for all \(a, b \in A\)

\[ d(a, b) \geq 0. \]

5. Prove that an ultrametric is a metric.