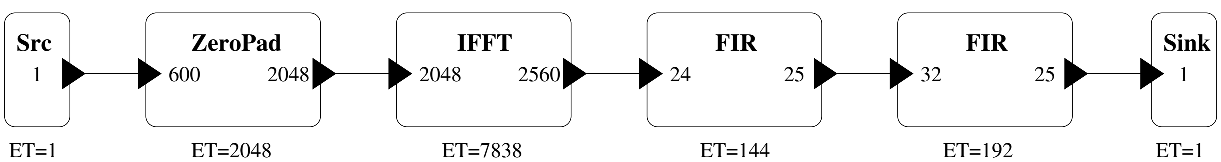
EE 144/244: Fundamental Algorithms for System Modeling, Analysis, and Optimization
Fall 2014

Dataflow – Timed SDF, Throughput Analysis

Stavros Tripakis
University of California, Berkeley

Main application of SDF: DSP hardware modeling

Orthogonal Frequency Division Multiple Access (OFDMA) application modeled in (**timed**) SDF:



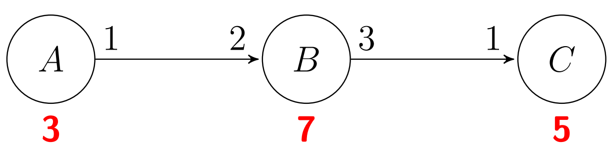
IFFT and FIR actors are configurable IPs from the Xilinx Coregen library, while the rest are user defined actors (source: National Instruments, e.g., see [Tripakis et al., 2011, Tripakis et al., 2012]).

Where do execution times come from?

- For existing hardware: measure clock cycles.
- To-be-synthesized hardware: estimate (harder problem).

TIMED SDF and THROUGHPUT ANALYSIS

Timed SDF



A Timed SDF Graph

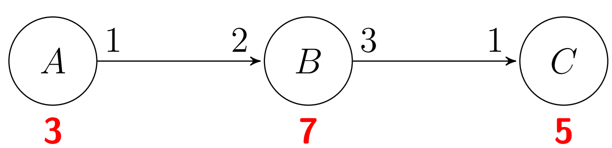
- **3, 7, 5**: execution times
  - how long it takes an actor to complete a firing
  - we will assume discrete time, can be extended to dense time

Timed SDF semantics

A Timed SDFG defines a **labeled transition system**:

- States record:
  - number of tokens in every channel;
  - number of instances of every actor running;
  - how much time is left for each instance until it completes its firing.
- Transitions: 3 types
  - a new instance begins firing;
  - an instance ends firing;
  - time passes.

Example



This timed SDFG defines the following LTS:

$$(g=(0,0), h=\{\}) \xrightarrow{begin\,A} (g=(0,0), h(A)=\{3\}) \xrightarrow{tick} (g=(0,0), h(A)=\{2\})$$

$$\xrightarrow{begin\,A} (g=(0,0), h(A)=\{3,3\}) \cdots$$

Timed SDF formal semantics

A Timed SDFG defines a **labeled transition system**:

- State = a tuple (g, h) containing
  - a vector g describing how many tokens are in every channel
  - a vector h describing
    - how many instances of each actor are running and
    - how much time is left until each instance completes its firing.
    - h associates to each actor A a *multiset* $h(A)$, i.e., a set which may contain multiple "copies" of the same element, e.g., $\{2, 2, 0\}$.
    - Why a multiset?
      $\Rightarrow$ could have multiple instances of the same actor active at the same time (parallelism, called *autoconcurrency* in SDF jargon).
- Initial state (unique):
  - $g(c)$ = number of initial tokens of channel c
  - $h(A) = \emptyset$ for all A (no instances running initially).

Timed SDF formal semantics

Transitions: of 3 possible forms

- $(g, h) \xrightarrow{begin A} (g', h')$: a new instance of A begins to fire.

  Precondition on g: enough tokens must be in the input queues of A.

  g': obtained by g by removing the corresponding tokens from the input queues of A.

  $h'(A)$: add to $h(A)$ the execution time of A.

- $(g, h) \xrightarrow{tick} (g, h')$: one time unit elapses.

  Precondition on h: there is no A such that $0 \in h(A)$ (this would mean one actor instance is ready to finish firing).

  h': decrement all actor instance execution times by 1.

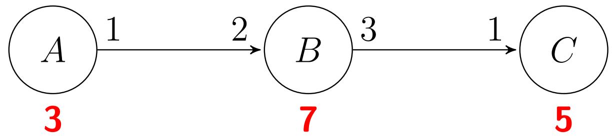
- $(g, h) \xrightarrow{end A} (g', h')$: one instance of A finishes firing.

  Precondition on h: $0 \in h(A)$.

  h': remove one 0 from $h(A)$.

  g': add to the output queues of A the tokens that A produces.

Example



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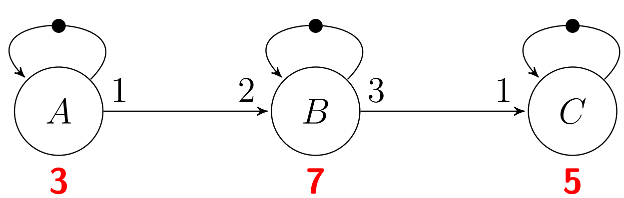
$$(g=(0,0), h=\{\}) \xrightarrow{begin\,A} (g=(0,0), h(A)=\{3\}) \xrightarrow{tick} (g=(0,0), h(A)=\{2\})$$

$$(g=(0,0), h(A)=\{3\}) \xrightarrow{begin\,A} (g=(0,0), h(A)=\{3,3\}) \cdots$$

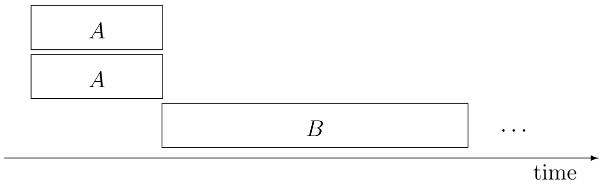
As with untimed SDF, this LTS is generally infinite.

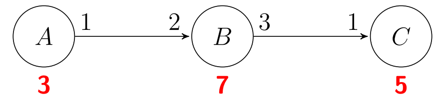
Forbidding autoconcurrency

How to forbid a new instance from starting until the previous one finishes?

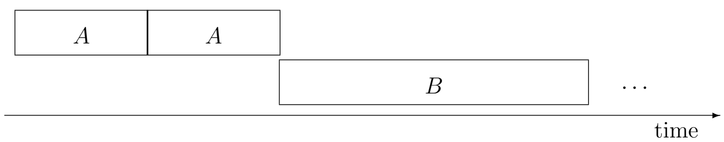


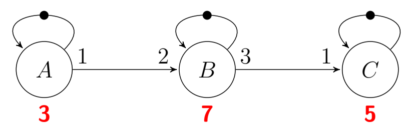
Pictorially





with autoconcurrency





without autoconcurrency

ASAP LTS ("self-timed execution")

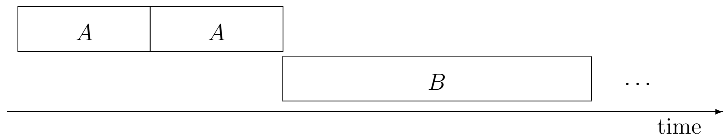
The *As Soon As Possible (ASAP)* LTS of a Timed SDFG is defined as before, with the additional rule that:

> *A $tick$ transition is allowed only if no $begin$ transitions are enabled.*[1]

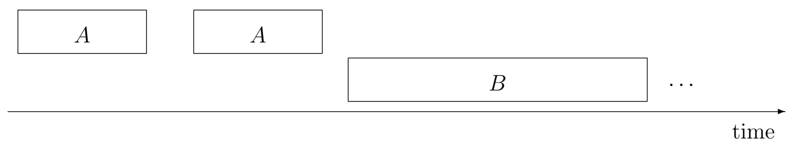
Justification: to estimate throughput, no need to delay actor firings unnecessarily.

[1] also if no end transitions are enabled, but that was already the case in the original semantics

Pictorially



ASAP execution

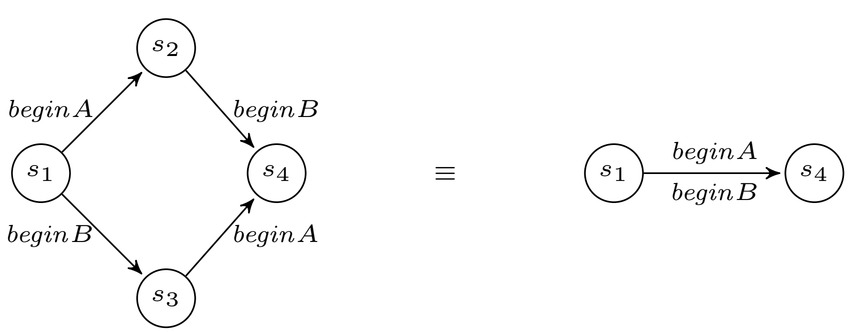


non-ASAP execution (with "slack")

Determinacy of ASAP execution

We can group together all non-$tick$ transitions in a single transition:

- Group everything that happens instantaneously.
- Deterministic! (as with Kahn networks – interleavings don't affect final result)

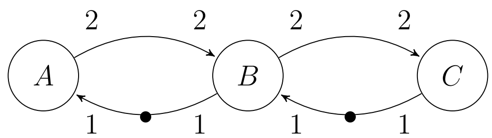


Similarly with end transitions.

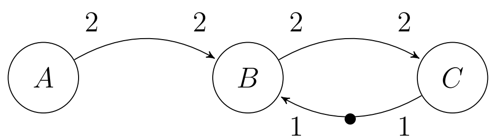
Strongly-connected (SDF) Graphs

Where every node (actor) can be reached by a directed path from every other node.

Strongly-connected:



Not strongly-connected:

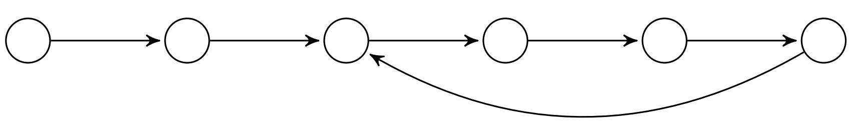


Determinacy and finiteness of ASAP execution

Theorem ([Ghamarian et al., 2006])

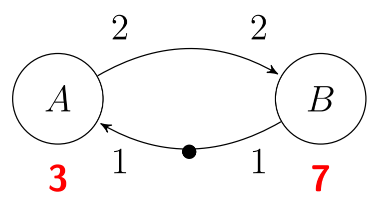
In consistent and strongly-connected timed SDFGs the deterministic ASAP execution is a **lasso** *(an initial segment followed by a cycle).*

A lasso:



Example: lasso of deterministic ASAP execution

Timed SDFG:



Deterministic ASAP execution:

$$(g=(0,1),h=\{\}) \xrightarrow{begin\,A} (g=(0,0),h(A)=\{3\}) \xrightarrow{tick} (g=(0,0),h(A)=\{2\}) \xrightarrow{tick}\xrightarrow{tick} (g=(0,0),h(A)=\{0\})$$

$$\xrightarrow{end\,A} (g=(2,0),h(A)=\{\},h(B)=\{\}) \xrightarrow{begin\,B} (g=(0,0),h(A)=\{\},h(B)=\{7\})$$

$$(g=(0,0),h(A)=\{\},h(B)=\{0\}) \xleftarrow{tick}\xleftarrow{tick}\xleftarrow{tick}\xleftarrow{tick}\xleftarrow{tick}\xleftarrow{tick}\xleftarrow{tick} (g=(0,0),h(A)=\{\},h(B)=\{7\})$$

$$(g=(0,0),h(A)=\{\},h(B)=\{0\}) \xrightarrow{end\,B} (g=(0,1),h=\{\})$$

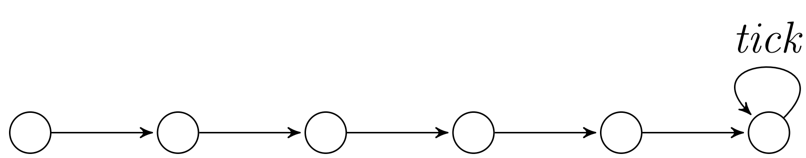
Determinacy and finiteness of ASAP execution

Theorem ([Ghamarian et al., 2006])

In consistent and strongly-connected timed SDFGs the deterministic ASAP execution is a **lasso** *(an initial segment followed by a cycle).*

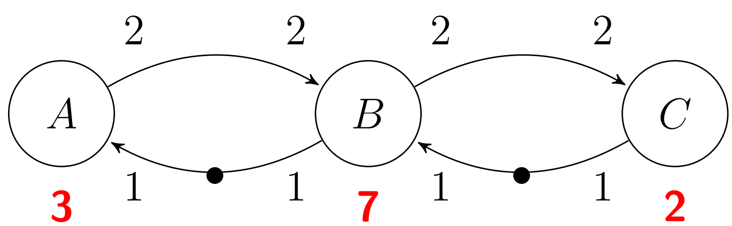
Does the theorem hold also in graphs that deadlock?

Yes: $tick$ transitions are always possible (time cannot be stopped).



Strongly-connected SDFGs ⇒ finite state-space and bounded queues

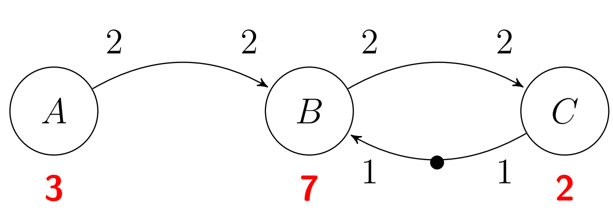
Strongly-connected ⇒ state space is finite ⇒ queues remain bounded.



What about non-strongly-connected SDFGs?

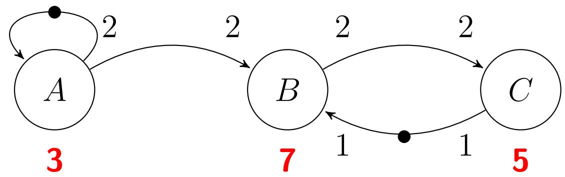
Non-strongly-connected SDFGs

A can fire arbitrarily many times in parallel (autoconcurrency) ⇒ queue from A to B grows unbounded ⇒ infinite state-space.



Non-strongly-connected SDFGs

What about timed SDFGs without autoconcurrency?



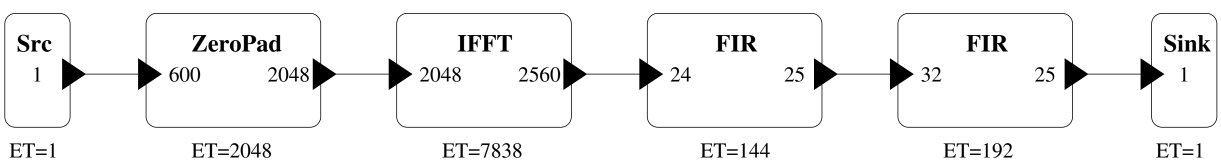
A's rate of firing (throughput) only depends on its execution time.

Rate of firing of B, C (downstream strongly-connected sub-graph) constrained by that of A (upstream strongly-connected sub-graph).

$\Rightarrow$ We will analyze throughput separately for each strongly-connected sub-graph.

What about this example?

The SDFG below is **not** strongly connected:

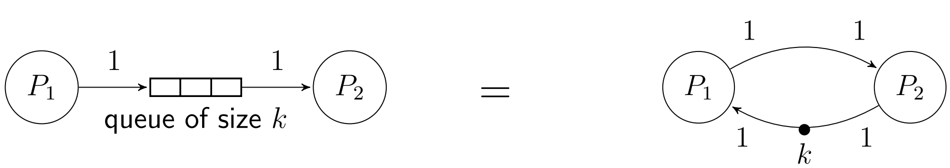


Does it mean we won't analyze throughput for this application?

In practice:

- Problem: optimize buffer size.
- Target throughput: 25 M samples/sec.

$\Rightarrow$ buffer size is an input to the problem $\Rightarrow$ finite queues $\Rightarrow$ strongly-connected graph.



THROUGHPUT ANALYSIS

Throughput

Throughput (in general) = average rate.

- e.g., communication networks: rate of data transmission

In our case:

- average rate of token productions (in given channel)
- average rate of actor firings (for given actor)
- these are multiples of each other:
  - let μ_A be the rate of firings of actor A
  - let μ_α be the rate of token productions in channel α
  - if A produces k tokens in α every time it fires, then

$$\mu_\alpha = k \cdot \mu_A$$

Actor Throughput

Let A be an actor in the timed SDFG of interest.
Let σ be an execution of the timed SDFG: σ is a trace in the corresponding LTS.

Define throughput of A in σ:

$$Th(A,\sigma) \quad \widehat{=} \quad \lim_{n\to\infty} \frac{\#\ endA \text{ transitions up to first } n\ ticks \text{ of } \sigma}{n}$$

Let $Th(A)$ denote the "best" (maximum) throughput:

$$Th(A) \quad \widehat{=} \quad \max_{\sigma} Th(A,\sigma)$$

Theorem ([Ghamarian et al., 2006])

$$\max_{\sigma} Th(A,\sigma) \quad = \quad Th(A, \textit{the ASAP execution})$$

Actor Throughput

Quiz: Consider an SDFG that deadlocks and let A be an actor in that SDFG.

What is the actor throughput $Th(A)$?

Does $Th(A)$ depend on A in this case?

Computing the Actor Throughput

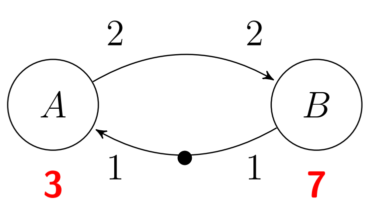
Let C denote the cycle of the ASAP execution lasso of the timed SDFG.

Theorem ([Ghamarian et al., 2006])

$$Th(A) \;=\; \frac{m_A}{m}$$

where m_A is the number of $endA$ transitions in C and m is the number of $tick$ transitions in C.

Example

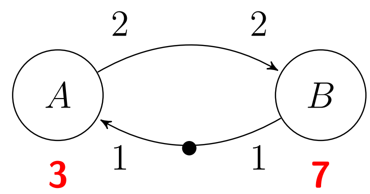


ASAP execution:

$$\begin{tikzcd}[column sep=small]
(g=(0,1), h=\{\}) \arrow[r, "beginA"] & (g=(0,0), h(A)=\{3\}) \arrow[r, "tick"] & (g=(0,0), h(A)=\{2\}) \arrow[r, "tick\ tick"] & (g=(0,0), h(A)=\{0\}) \arrow[d, "endA"] \\
 & & & (g=(2,0), h(A)=\{\}, h(B)=\{\}) \arrow[d, "beginB"] \\
 & (g=(0,0), h(A)=\{\}, h(B)=\{0\}) \arrow[uul, "endB"] & & (g=(0,0), h(A)=\{\}, h(B)=\{7\}) \arrow[ll, "tick\ tick\ tick\ tick\ tick\ tick\ tick"']
\end{tikzcd}$$

$$Th(A) = Th(B) = \frac{1}{10}$$

Couldn't we do this more easily?



1. Compute periodic schedule (while checking for deadlocks)

$$(AB)^\omega$$

2. Compute total duration of schedule

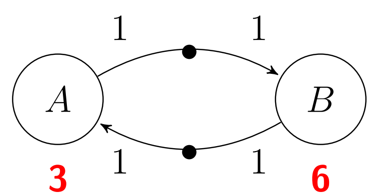
$$ET(A) + ET(B) = 3 + 7 = 10$$

3. Compute throughput

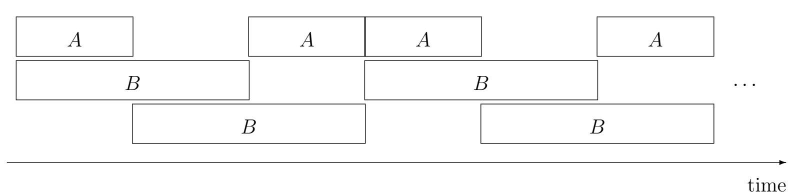
$$Th(A) = \frac{\#\text{ times } A \text{ appears in schedule}}{10} = \frac{1}{10}$$

Does this method work for all SDFGs?

Parallelism and pipelining!



The simplistic method of the previous slide gives:
$ET(A) + ET(B) = 9$, therefore, estimates throughput to be $\frac{1}{9}$.
The ASAP execution takes pipelining into account:



and computes the correct (maximal, assuming parallelism) throughput:

$$Th(A) = Th(B) = \frac{2}{9}$$

Bibliography

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