Interchange Semantics For Hybrid System Models

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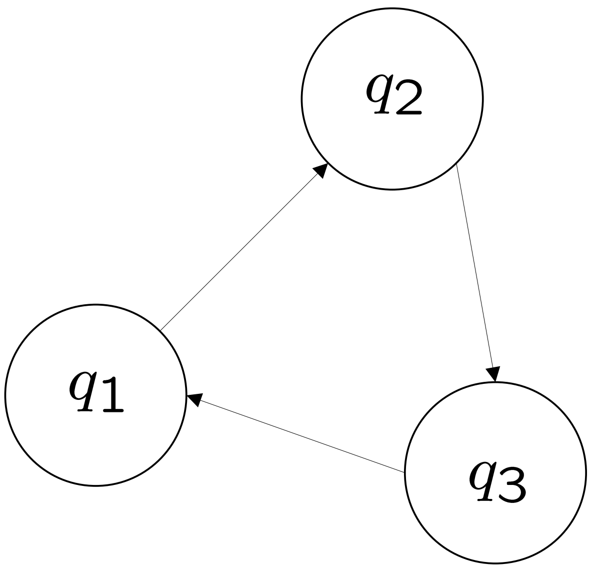
Hybrid systems

A hybrid system is a tuple

$$\mathcal{H} = (\mathbf{Q}, \mathbf{U}_D, E, X, U, V, \mathcal{S}, Inv, R, G)$$



$$\mathcal{S}(q_1) = (\dot{x} = f(x, u, t))$$

$$G(q_1, q_2) = (x_1 \geq x_2 \wedge v_1 > v_2)$$

$$Inv(q_1) = (\bar{G}(q_1, q_2))$$

$$R(q_1, q_2) = (x' = r(x, u))$$

Hybrid system semantics

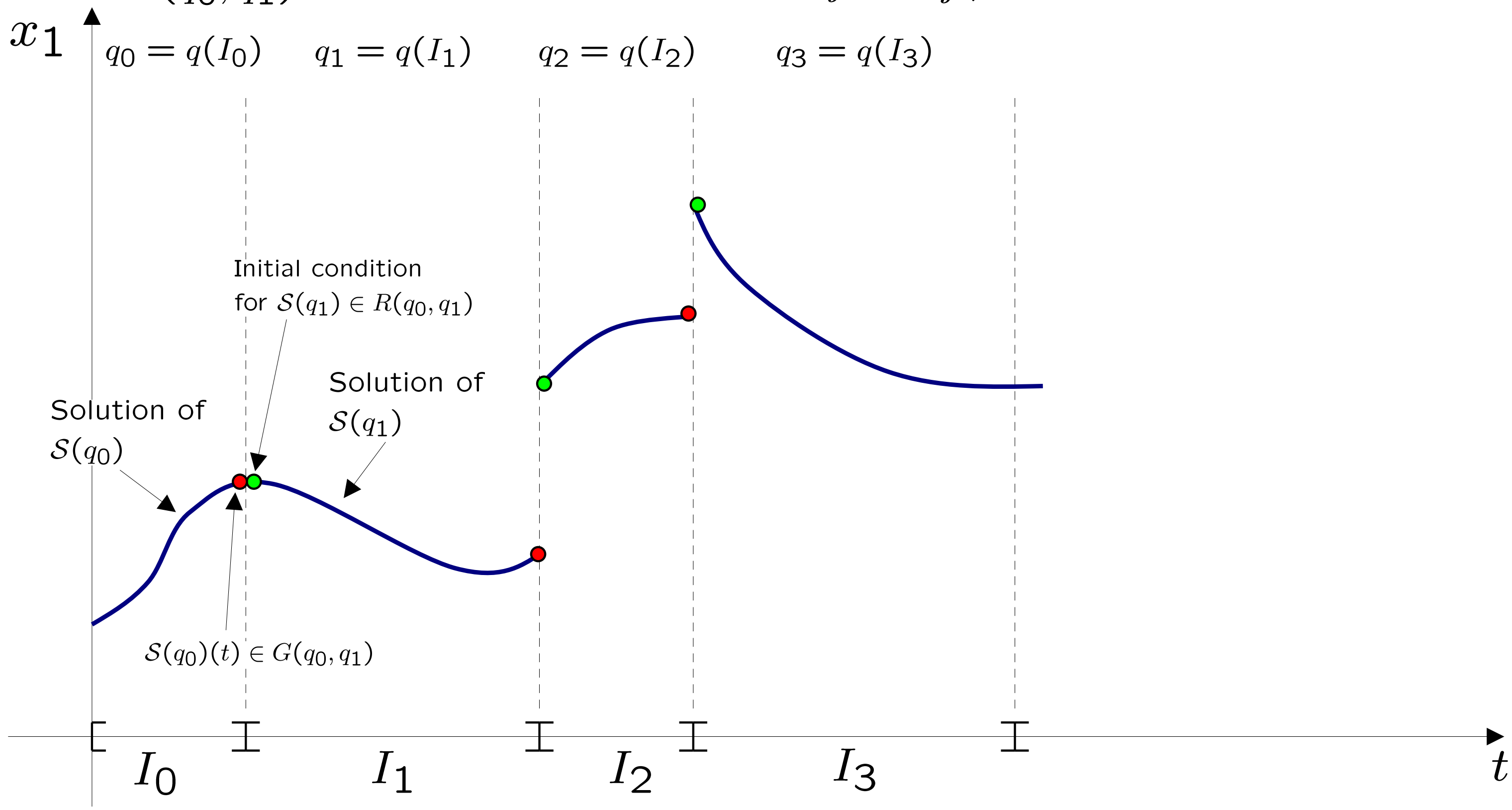
A hybrid time basis τ is a finite or an infinite sequence of intervals

$$I_j = \{t \in \mathbb{R} : t_j \leq t \leq t'_j\}$$

where $t_j \leq t'_j$ and $t'_j = t_{j+1}$

Hybrid systems execution

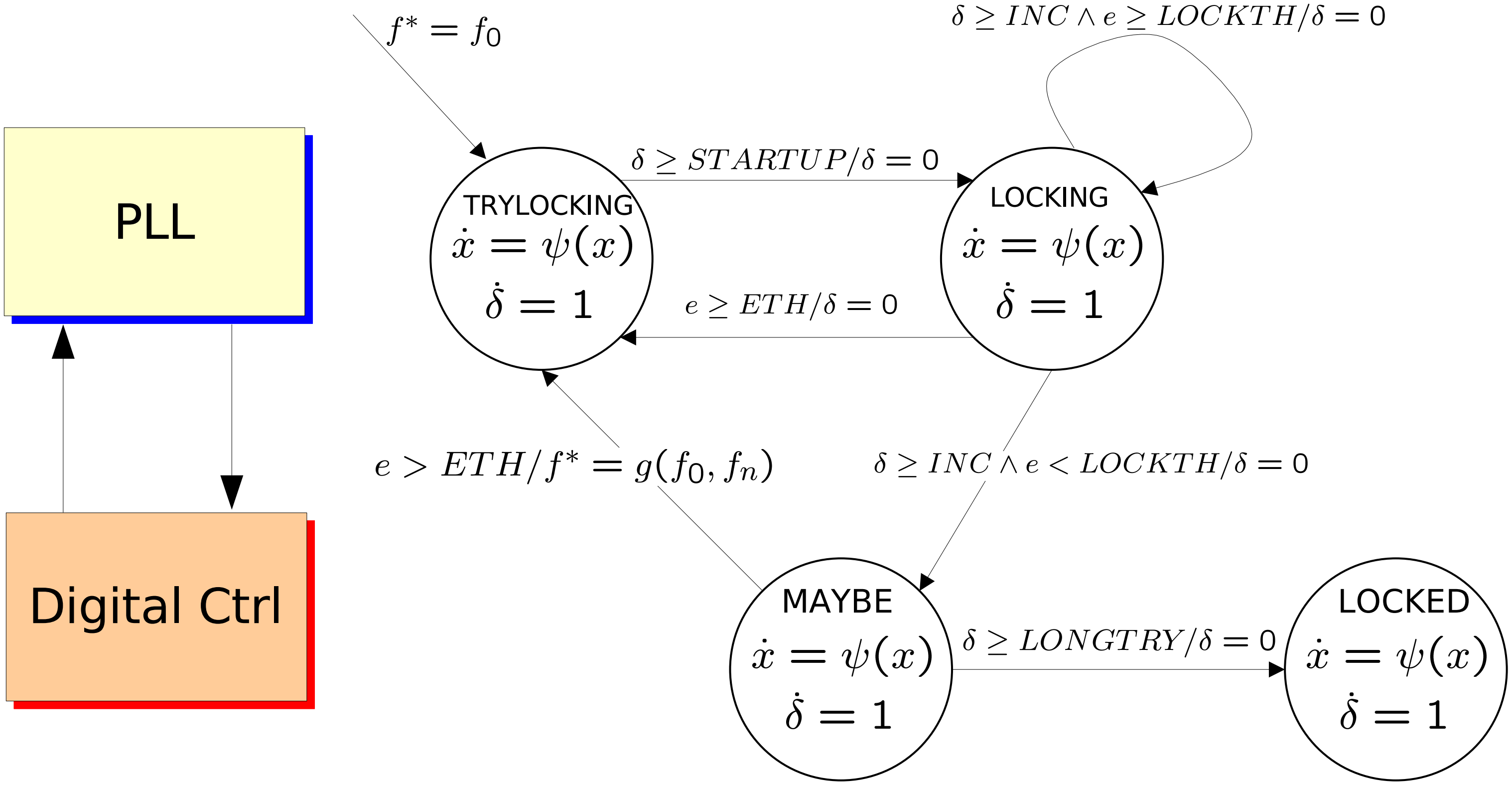
$(q_0, q_1) \in E$ $\qquad$ $(q(I_j), q(I_{j+1})) \in E, \quad \forall j$



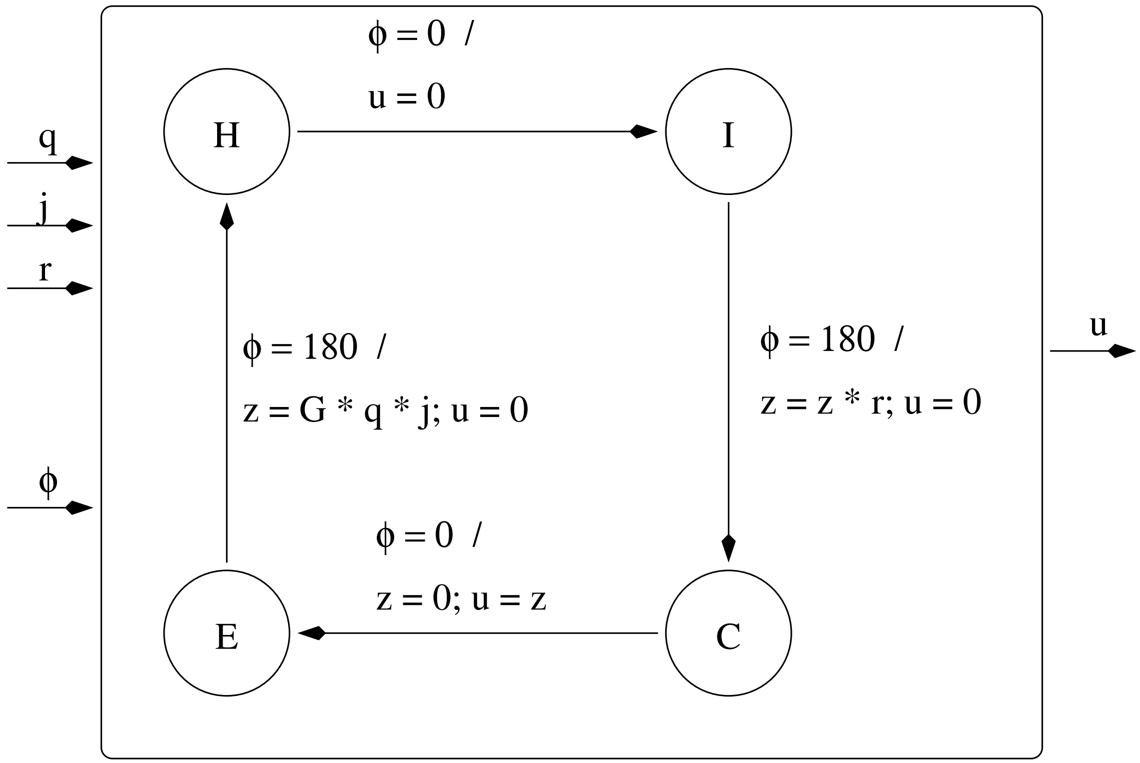
Applications

- Abstract uninteresting dynamics

Digitally controlled PLL



A car engine



Need for an interchange format

- Hybrid Systems (HS) have proven to be a powerful design representation for system -level design.
- There has been a proliferation of tool for simulation, verification and synthesis of HS but...
- ... all based on different models.
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- We presented a proposal for an IF (HSCC2005).
- We have defined its semantics (HSCC2006).
- Here we summarize our findings and justify our choices.

Outline

1. Interchange Formats in EDA
2. Interchange Format Syntax and Abstract Semantics
3. Composition and Hierarchy
4. Conclusions

Interchange Formats in EDA

Interchange Formats in EDA: a Brief History

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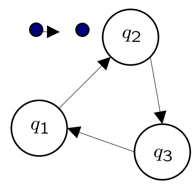
  - Not flexible. It reduces the degree of freedom of the tools that share the date using the format.
- An interchange format has a formal *abstract semantics* that can be refined into *concrete semantics*
  - An interchange format must be capable of capturing the largest possible class of models in use today and even tomorrow
  - At the same time has to have precise semantics to avoid ambiguity.

The Big Picture

$$\mathcal{H} = (\mathbf{Q}, \mathbf{U}_D, E, X, U, V, \mathcal{S}, Inv, R, G)$$

HS-Denotational Description



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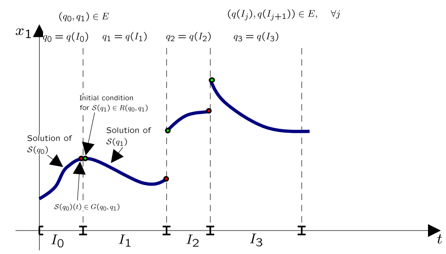
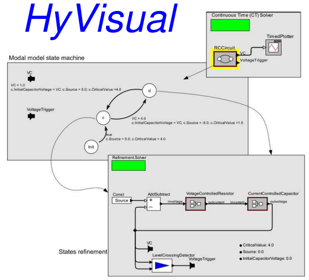
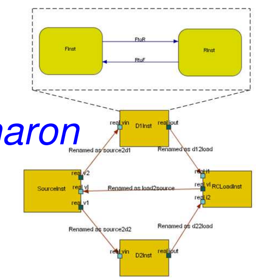


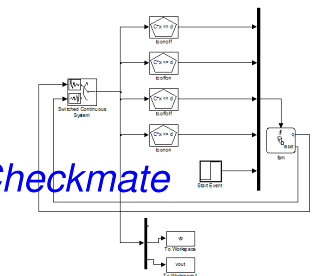
HyVisual



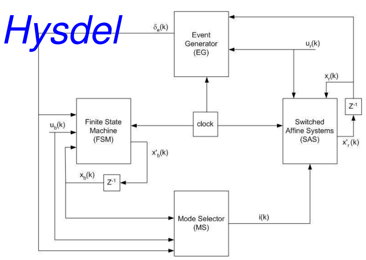
Charon

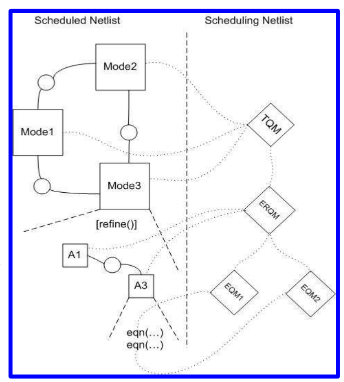


Checkmate



Hysdel





Interchange Format

Interchange Format Syntax and Abstract Semantics

Preliminary Definitions

Valuations of variables : Given a variable with name v, its value is denoted by $val(v)$.

Valuation of tuples of variables : If V is the tuple $(v_1, ..., v_n)$ then $val(V) = (val(v_1), ..., val(v_n))$.

Valuation of sets of variables : If V is the set $\{v_1, ..., v_n\}$ then its valuation is the multi-set $val(V) = \{val(v_1), ..., val(v_n)\}$.

Valuations domain : For a set of variables V, the set of all possible valuations of V is denoted by $\mathcal{R}(V)$.

Lifting : Given a subset $D \subseteq \mathcal{R}(V)$ and $V' \supseteq V$, the lifting of D to V' is given by the operator $\mathcal{L}(V')(D) = \{p' \in \mathcal{R}(V') : p'|_V \in \mathcal{R}V\}$.

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- $V_t = \{v_{t1}, ..., v_{tn}\}$ is a set of temporary variables,
- $\pi : E \longrightarrow \{1, 2, \ldots, |E|\}$ is an equation ordering function.

Syntax Example

A bouncing ball can be modeled as a hybrid system with $V = \{y, v\}$, $E = \{\dot{v} = -g, \dot{y} = v\}$. There are two domains: $D_1 = \{\{val(y), val(v)\} : val(y) \leq 0 \wedge val(v) < 0\}$ and $D_2 = \{\{val(y), val(v)\} : val(y) > 0\}$, hence $\mathcal{D} = \{D_1, D_2\}$; $I = \{1\}$, $\sigma(D_1) = \sigma(D_2) = \{1\}$, $\omega(1) = E$. The reset function is defined as follows: $\rho(D_2, D_1, val(V)) = \{val(y), -\epsilon val(v)\}$.

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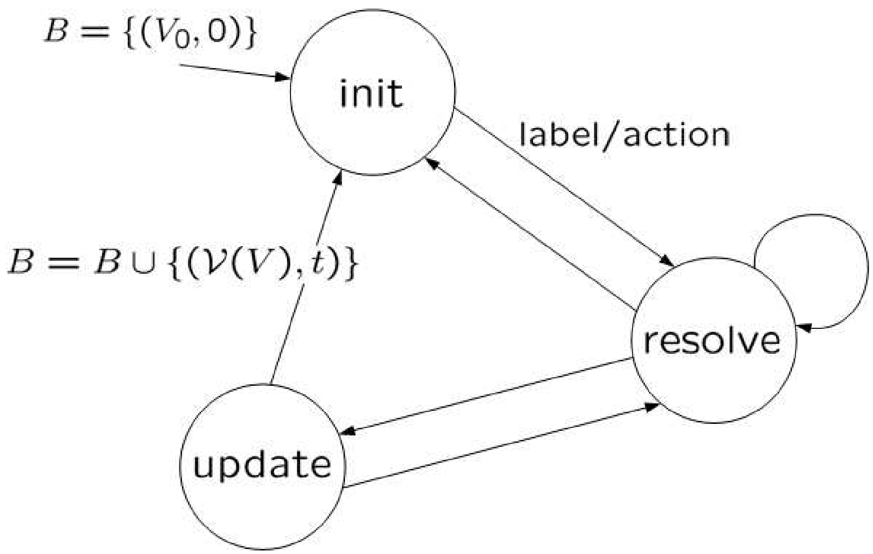
- T is a time stamper:
  - selects the next time stamp,
  - decides whether a new pair (val, t) can be added to the set B;
- `resolve`, `init` and `update` are three algorithms.

Time Stamper Automaton



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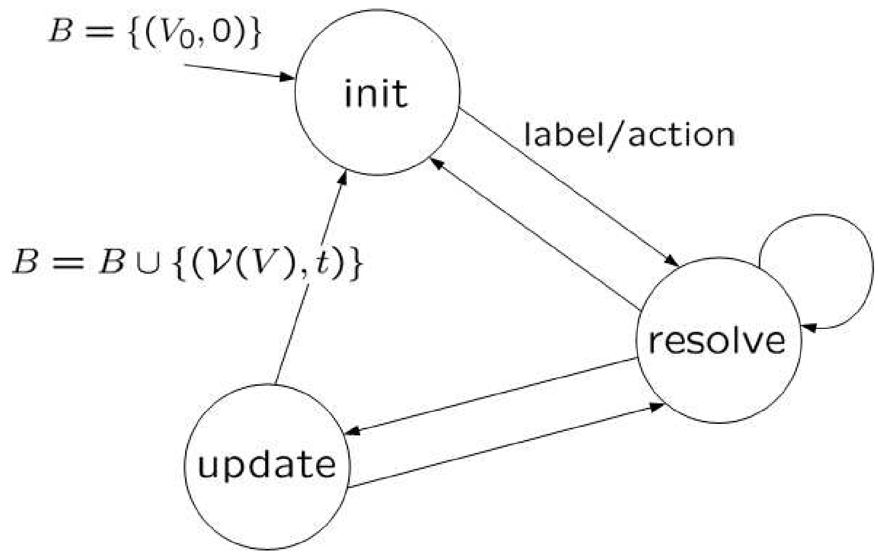

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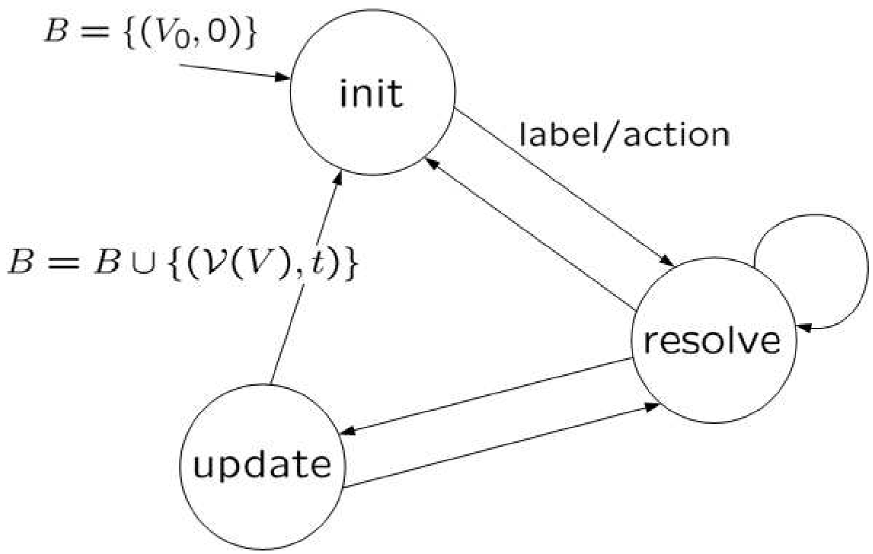

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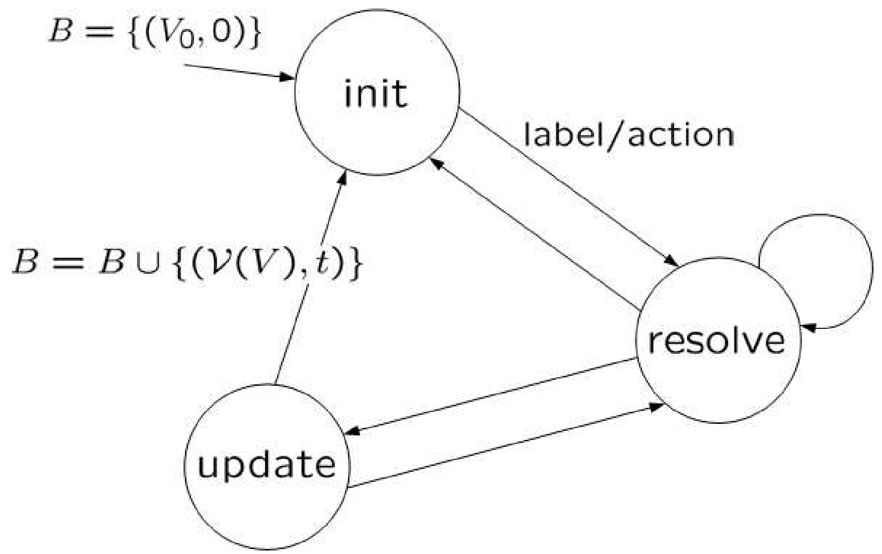

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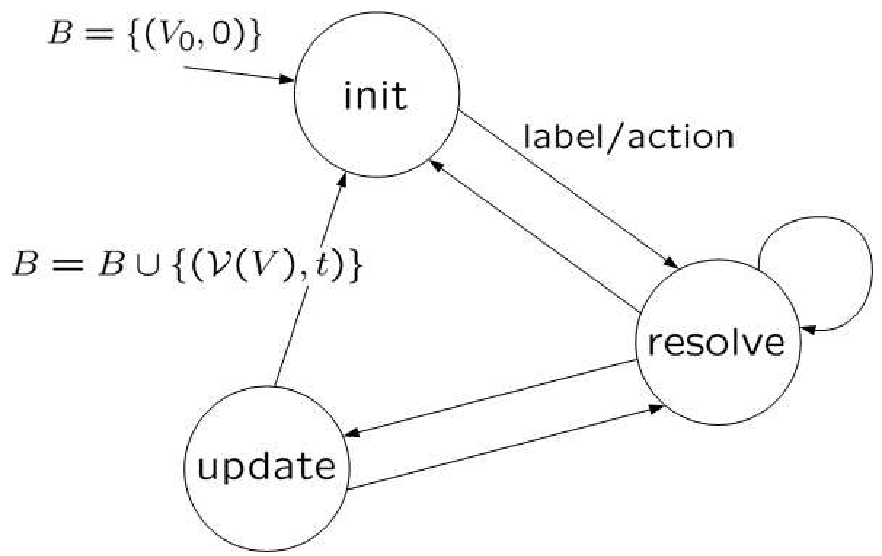

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- Set of actions: `next`, `resolve`, `init`, `update`.
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- Valuation and time stamp acceptance: $B = B \cup (\mathcal{V}(V), t)$.

Resolve Algorithm

resolve(t)

$\mathcal{D}' \Leftarrow \{D \in \mathcal{D} \mid val(V_t) \in D\}$ *// Compute the set of active domains.*

$I \Leftarrow \emptyset,\ E_t \Leftarrow \emptyset$

$I \Leftarrow \cup_{D \in \mathcal{D}'} \sigma(D)$ *// Collect all active dynamics and components.*

for all $i \in I$ **do**

  $E_t = E_t \cup \omega(i)$ *// Collect all active equations.*

end for

$\texttt{sort}(E_t, \pi)$ *// Order the equations.*

for all $e_i \in E_t$ **do**

  $\texttt{solve}(e_i, t)$

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$\mathcal{D}'' \Leftarrow \{D \in \mathcal{D} \mid val(V_t) \in D\}$ *// Set of active domains after the computation.*
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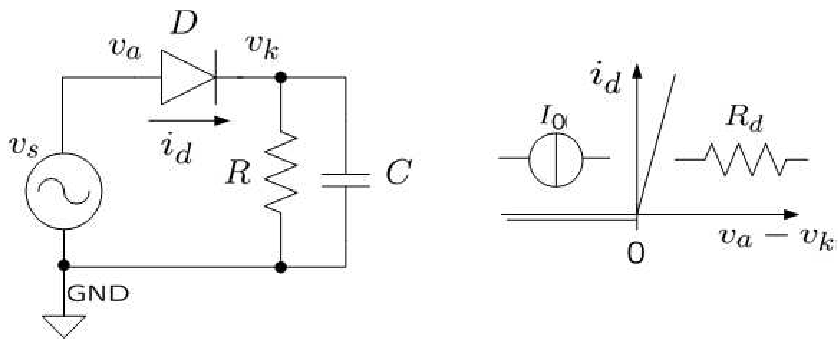
Refinement into Concrete Semantics

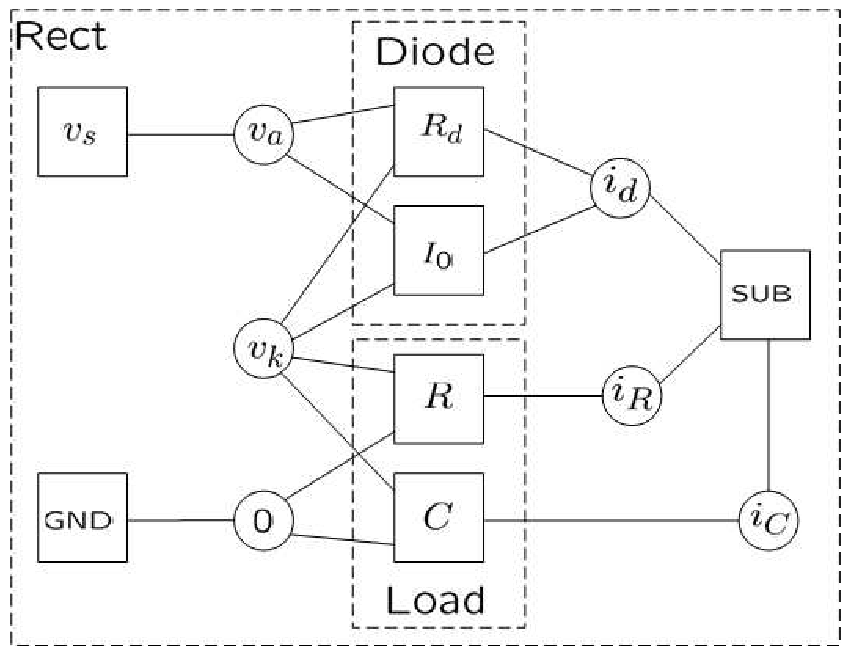
- Define the time stamper automaton: conditions and actions
  - Multiple iterations for fixed point or event detection
- Define the `next` function
  - Different algorithm to decide the next time stamp
- Define the `domainchange` function
  - Different transition semantics
- Define the `solve` function
  - Different integration methods

Composition and Hierarchy

Running Example



(a) Half-wave rectifier



(b) Block diagram of the half-wave rectifier

Composition of Hybrid Systems

Given $H_1 = (V_1, V_{t1}, E_1, \mathcal{D}_1, I_1, \sigma_1, \omega_1, \rho_1, \pi_1)$ and $H_2 = (V_2, V_{t2}, E_2, \mathcal{D}_2, I_2, \sigma_2, \omega_2, \rho_2, \pi_2)$, $H = H_1 \mid\mid H_2$ is such that:

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- $V = V_1 \cup V_2,\ V_t = V_{t1} \cup V_{t2},\ E = E_1 \cup E_2,\ \mathcal{D} = \mathcal{L}(V)(\mathcal{D}_1) \cup \mathcal{L}(V)(\mathcal{D}_2)$
- $I = \{1, ..., |I_1| + |I_2|\}$
- $\forall D \in 2^{\mathcal{R}(V)},\ \sigma(D) = \sigma_1(D|_{V_1}) \cup (\sigma_2 + |I_1| + 1)(D|_{V_2})$ where $(\sigma + k)(D) = \{n + k : n \in \sigma(D)\}$ is a shifting of the indexes;

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- $$\begin{array}{ll} \omega(i) = \omega_1(i), & \text{if } 1 \leq i \leq |I_1|, \\ \omega(i) = \omega_2(i - |I_1|), & \text{if } |I_1| + 1 \leq i \leq |I_1| + |I_2| \end{array}$$

Composition of Hybrid Systems

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- $\pi(e) = \begin{cases} \pi_1(e) & if\ e \in E_1 \\ \pi_2(e) + |I_2| + 1 & if\ e \in E_2 \end{cases}$

Composition of Hybrid Systems

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- $\pi(e) = \begin{cases} \pi_1(e) & if\ e \in E_1 \\ \pi_2(e) + |I_2| + 1 & if\ e \in E_2 \end{cases}$
- $$\begin{array}{ll} \rho(D_i, D_j, val(V)) = & \mathcal{L}(V)(\rho_1(D_i|_{V_1}, D_j|_{V_1}, val(V_1)) \cup \\ & \mathcal{L}(V)(\rho_2(D_i|_{V_2}, D_j|_{V_2}, val(V_2)) \end{array}$$

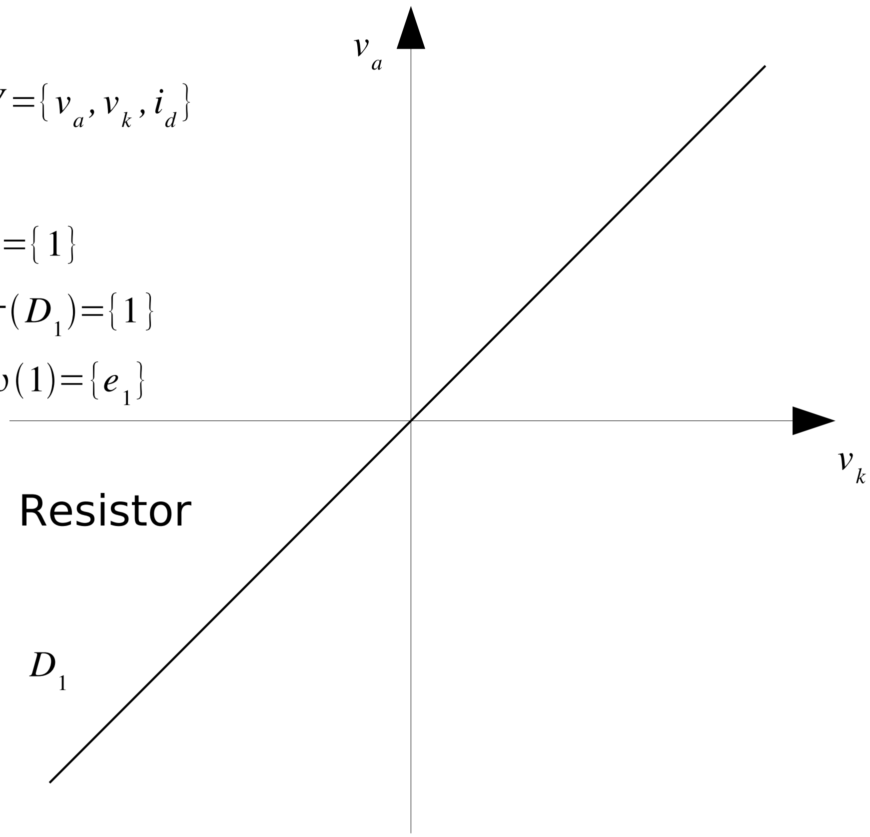
Composition Example

$V = \{v_a, v_k, i_d\}$

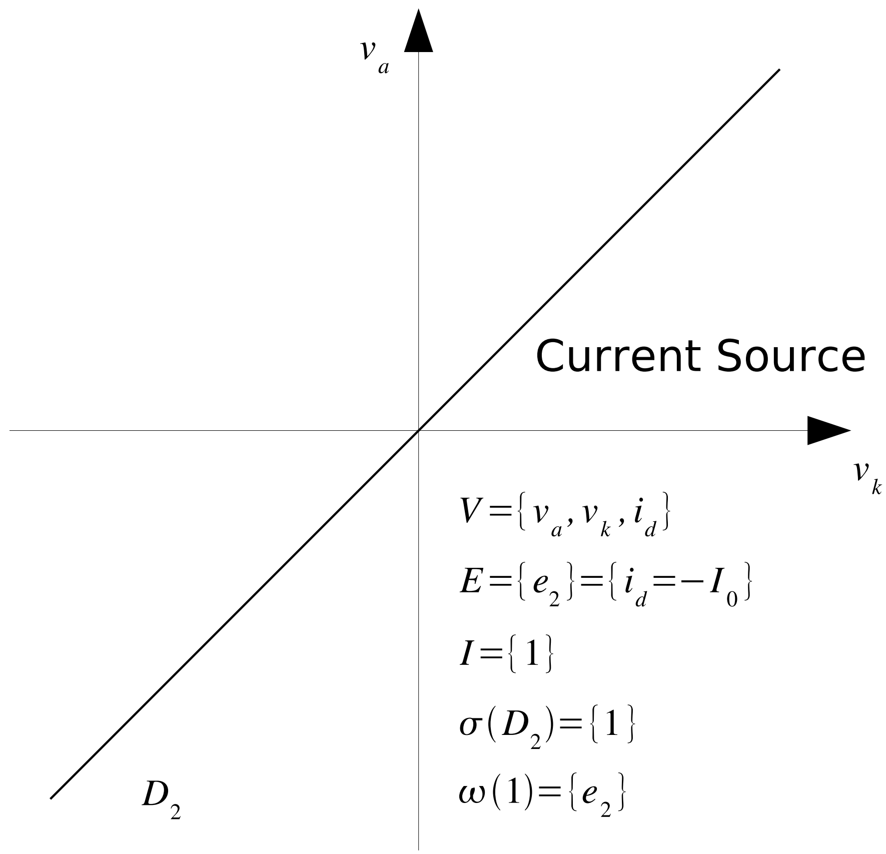
$I = \{1\}$

$\sigma(D_1) = \{1\}$

$\omega(1) = \{e_1\}$



Composition Example



$$V=\{v_a, v_k, i_d\}$$

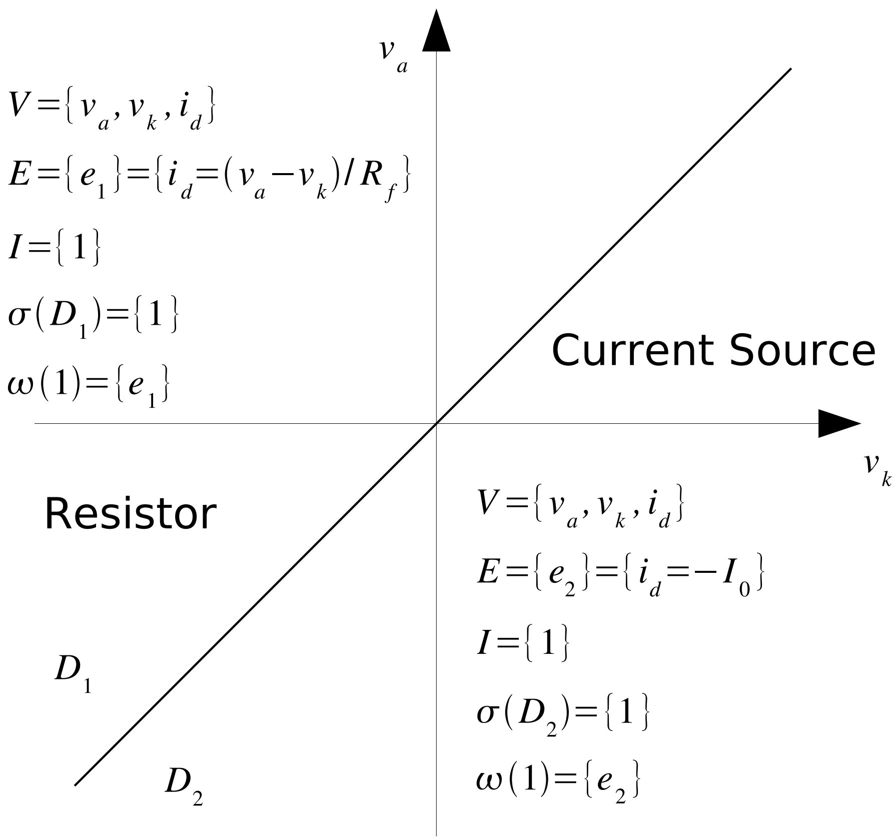
$$E=\{e_2\}=\{i_d=-I_0\}$$

$$I=\{1\}$$

$$\sigma(D_2)=\{1\}$$

$$\omega(1)=\{e_2\}$$

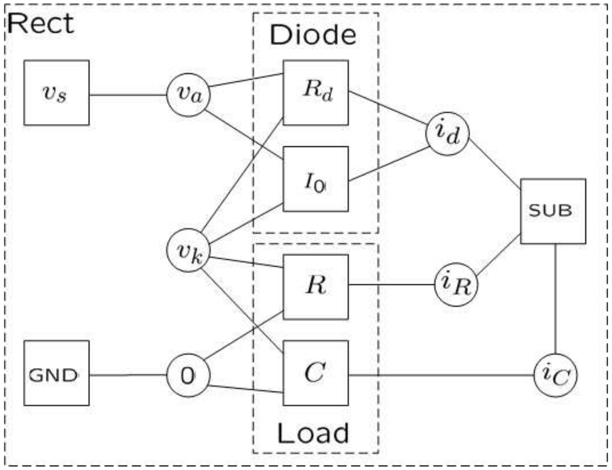
Composition Example



$diode = R_d \mid\mid I_d$

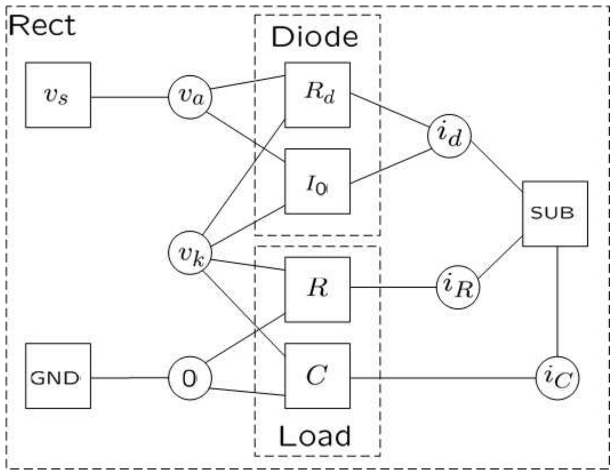
- $diode.V = \{v_a, v_k, i_d\}$,
- $diode.E = \{e_1, e_2\}$,
- $diode.\mathcal{D} = \{D_1, D_2\}$,
- $I = \{1, 2\}$,
- $diode.\sigma(D_1) = \{1\}$,
  $diode.\sigma(D_2) = \{2\}$,
- $\omega(1) = e_1$, $\omega(2) = e_2$,

Representing Hierarchy



(c) $Rect = v_s \,||\, (R_d \,||\, I_0) \,||\, \mathrm{GND} \,||\, \mathrm{SUB} \,||\, (R \,||\, C)$

Representing Hierarchy



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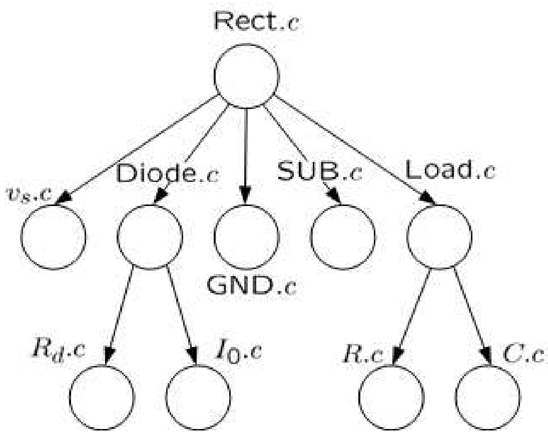
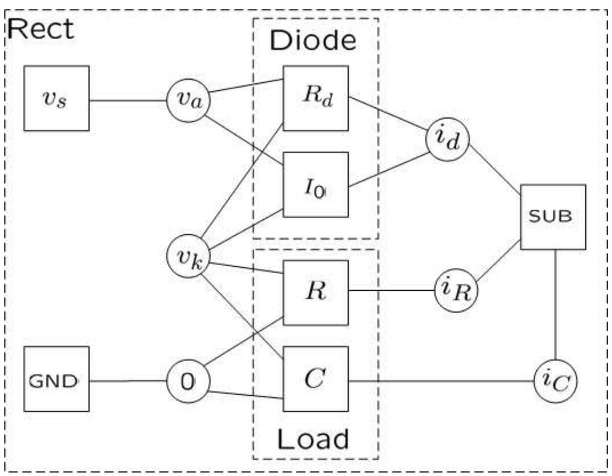


(d) Component Tree

Representing Hierarchy



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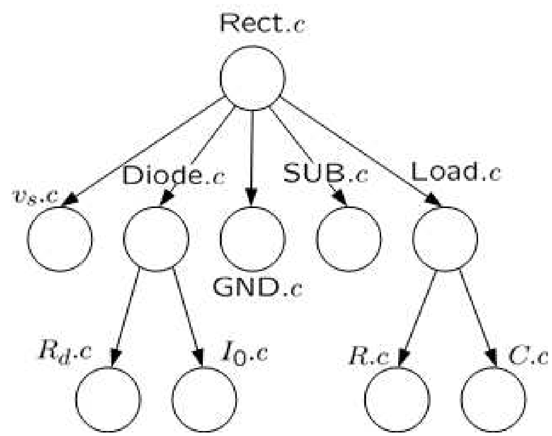

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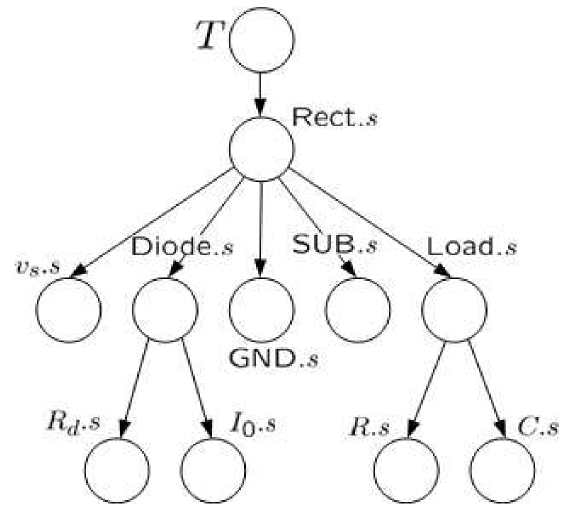
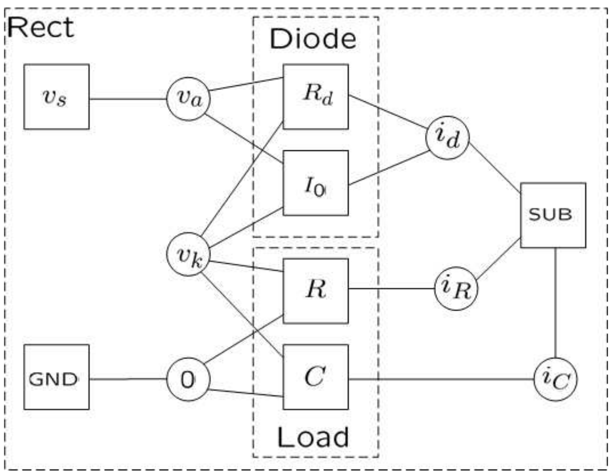


(e) Scheduler Tree

Representing Hierarchy



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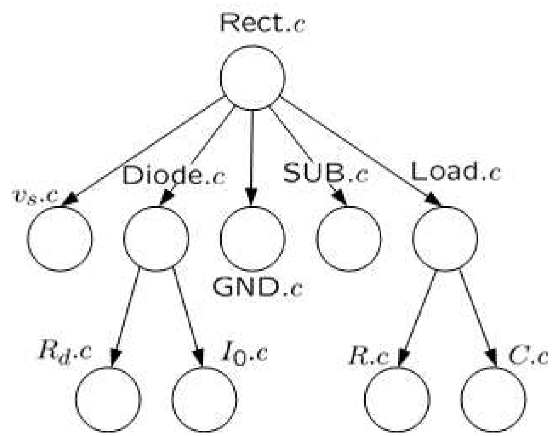
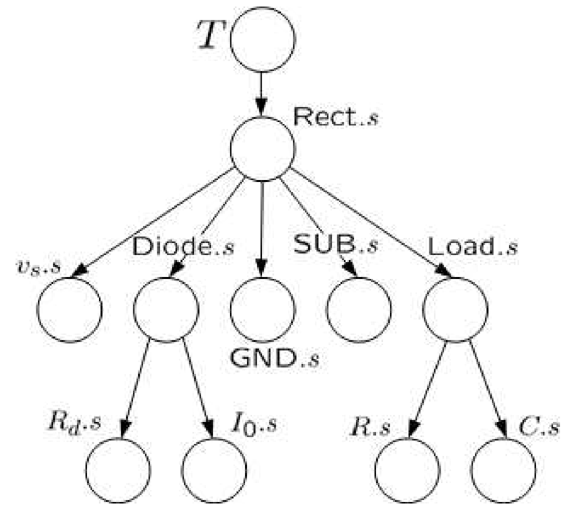


(d) Component Tree



(e) Scheduler Tree

Let $\mathcal{G} : S_N \longrightarrow 2^{S_N}$ be a function that associates to each scheduler the set of its children, and let $\Pi : S_N \longrightarrow \{1, \ldots, |S_N|\}$ be a global ordering of the nodes. Let $\mathcal{I} : \mathcal{C} \longrightarrow \mathcal{S}$ be a function that associates to each component it's scheduler.

Scheduler's `resolve` Algorithm

resolve(t)
$children \Leftarrow \mathcal{G}(s)$
if $children = \emptyset$ **then**
  // s is a leaf, proceed to solve the equations and end recursion
  $\mathcal{D}' \Leftarrow \{D \in \mathcal{I}^{-1}(s).\mathcal{D} \mid val(\mathcal{I}^{-1}(s).V_t) \in D\}$
  $J \Leftarrow \cup_{D \in \mathcal{D}'} s.\sigma(D)$
  $E_t \Leftarrow \cup_{i \in J} s.\omega(i)$
  $E_t \Leftarrow \texttt{sort}(E_t, s.\pi)$
  for all $e_i \in E_t$ **do**
    $\texttt{solve}(e_i, t)$
  end for
  `markchange` ($\mathcal{D}', val(\mathcal{I}^{-1}(s).V_t)$)
else
  // s is not a leaf, continue the recursion
  $children \Leftarrow \texttt{sort}(children, \Pi)$
  for all $s_i \in children$ **do**
    $s_i.\texttt{resolve}(t)$
  end for
end if

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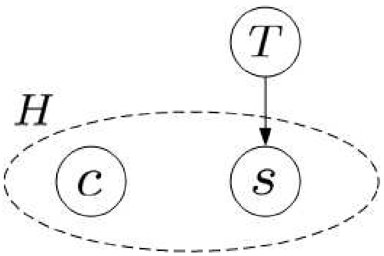
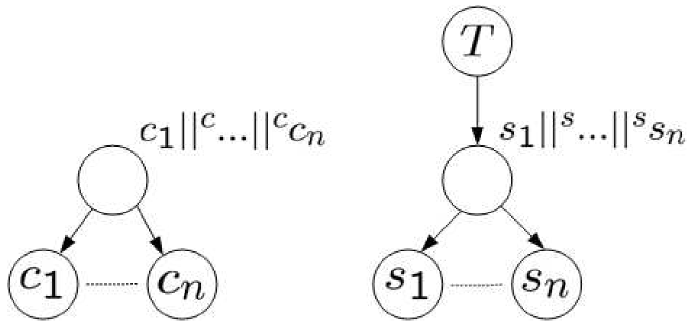
Examples and Conclusions

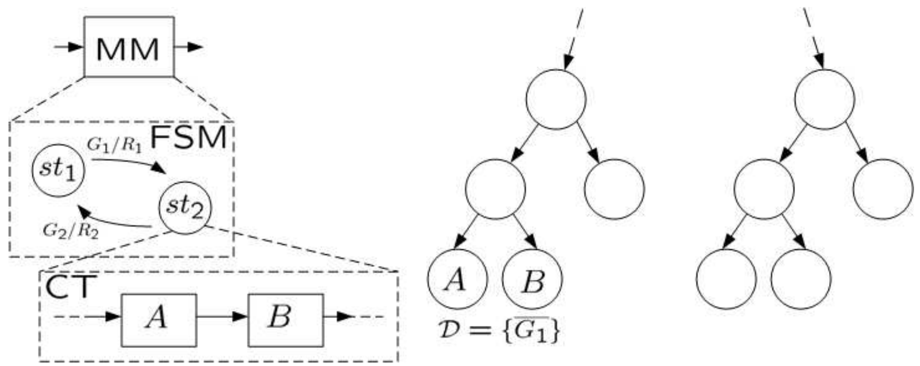
Examples



Structure of CHECKMATE programs



Structure of HYTECH programs



Structure of HYVISUAL programs

Conclusions and Future Work

- We have presented an abstract semantics for hybrid systems. It can be refined by specifying:
  - the time stamper automaton
  - the functions `domainchange`, `solve`, `next`
- We have shown how the structure of hybrid systems can be captured in the interchange semantics.
- We have implemented a prototype of the half-wave rectifier in METROPOLIS and can be downloaded at `http://embedded.eecs.berkeley.edu/hyinfo`.

- Future work:
  - Implementation of a METROPOLIS library for the interchange format;
  - Integration of a DAE solver (in collaboration with Jaijeet Roychowdhury, University of Minnesota)
  - Implementation of translators to/from known tools