Update on the theory

Jerry Burch

Roberto Passerone

Alberto Sangiovanni-Vincentelli

Objectives and outline

- Provide the foundation to represent different semantic domains for the Metropolis intermediate format
- Study the problem of heterogeneous interaction
- Formalize concepts such as abstraction and refinement

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An example of interaction

- Combine a synchronous model with a dataflow model
- Synchronous model
  - Total order of event
- Data flow model
  - Partial order of events
- Discrete Time model
  - Metric order of events

J. Burch, R. Passerone, A. Sangiovanni-Vincentelli, "Overcoming Heterophobia: Modeling Concurrency in Heterogeneous Systems", *to appear in Proceedings of the International Conference on Application of Concurrency to System Design,* Newcastle upon Tyne, U.K., June 2001.

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An example of heterogeneous interaction

- The interaction is derived from a common refinement of the heterogeneous models
- The resulting interaction depends on the particular refinements employed
- Our objective is to derive the consequences of the interaction at the higher levels of abstraction

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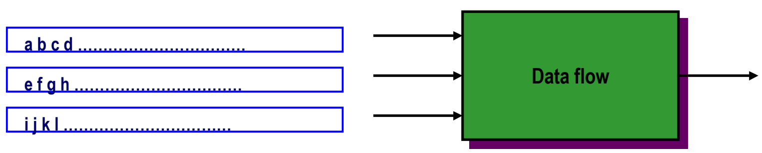
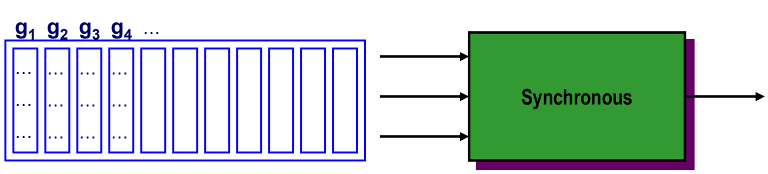
Data flow model

- Assume signals take values from a set V
- Each signal is a sequence from V (an element of V*)
- Let A be the set of signals
- One behavior is a function
  - $f : A \rightarrow V^*$
- An data flow agent is a set of those behaviors



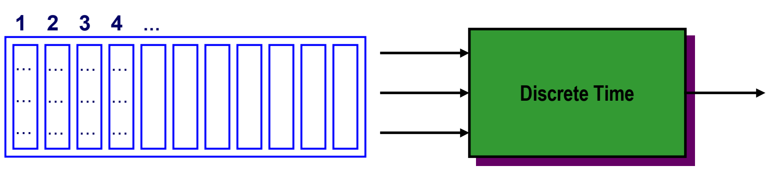
Synchronous model

- Signals are again sequences from V (elements of V*)
- But are synchronized
- One element of the sequence is $g : A \rightarrow V$
- One behavior is a sequence of those functions
  - $<g_i> \in (A \rightarrow V)^*$
- A synchronous agent is a set of those sequences



Discrete Time model

- Assume time is represented by the positive integers N
- Then define a behavior
  - h: N → (A → V)
- A discrete time agent is a set of those functions



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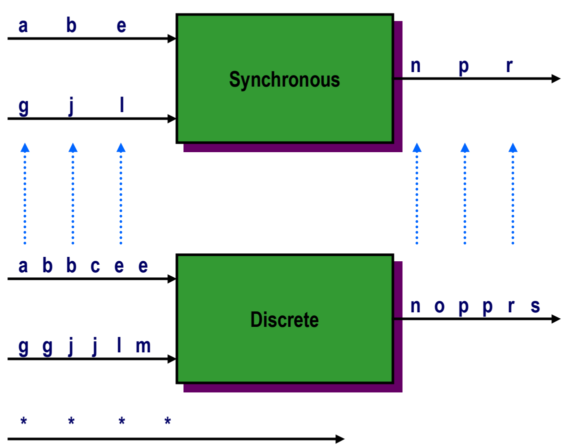
Discrete to Synchronous abstraction



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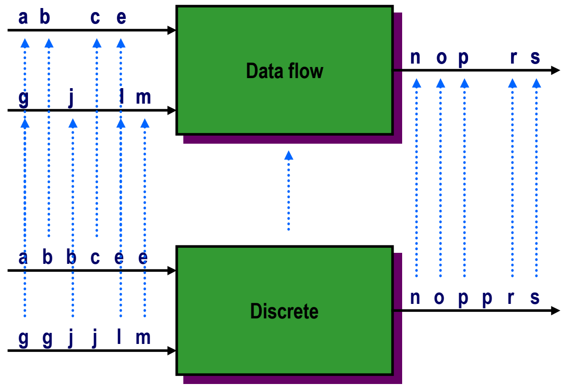
Discrete to Data flow abstraction



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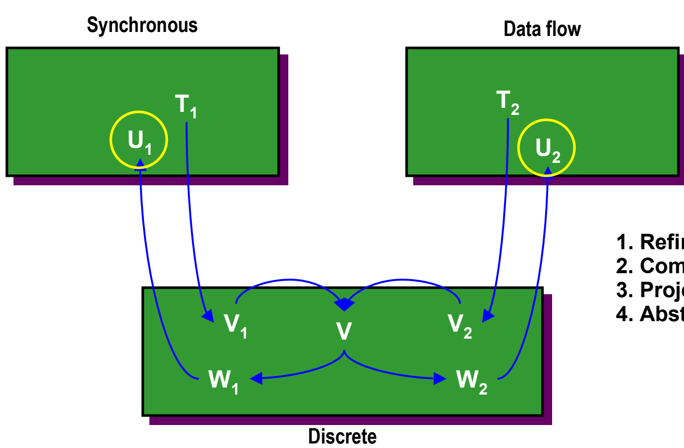
Interaction Propagation



1. Refinement
2. Composition
3. Projection
4. Abstraction

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Key points

- The outlined technique defines the effects of the interaction
- The result depends on
  - The notion of composition at the refined level
  - The particular abstraction and refinement
- We can't define the interaction uniquely!
  - Note: both Synchronous and Data flow are untimed sequences
  - Could just have equated them
- How can we generalize? Need a formal approach to this problem

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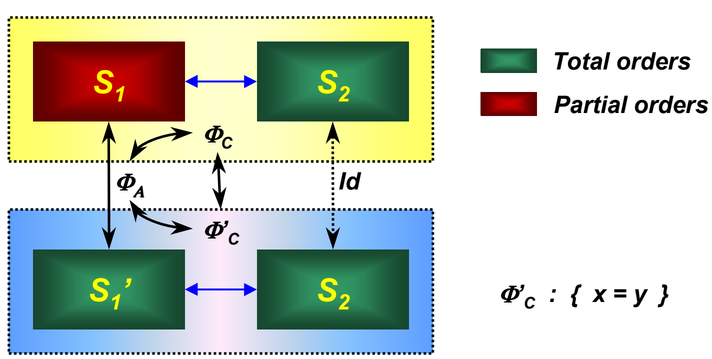
Maintaining consistency across refinement



Φ'_C : { $x = y$ }

Φ_C, Φ_A and Φ'_C must be consistent, together with the specifications

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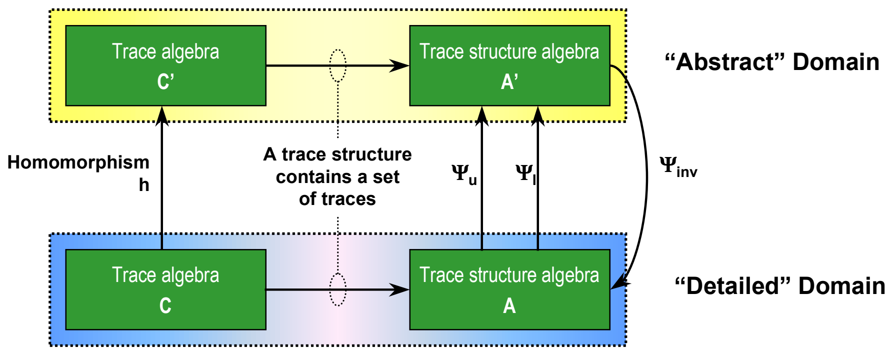
Trace algebras and Trace Structures algebras



Let T_{spec} and T_{impl} be trace structures in A. Then

if $\Psi_u(T_{impl}) \subseteq \Psi_l(T_{spec})$ then $T_{impl} \subseteq T_{spec}$

Trace Algebra

- Let *W* be a set of signals and *A* a subset of *W*
- A trace algebra is a set of traces, each taking symbols from A, with operations of projection and renaming
- Formally, a trace algebra C_C is a triple (B_C, *proj*, *rename*) where
  - for each *A*, $B_C(A)$ is a non-empty set, called the set of traces over *A*. Let B_C also be the set of all traces, i.e. the union over all subsets *A* of *W* of the traces $B_C(A)$.
  - for $B \subseteq A$, *proj*(*B*) is a function from B_C to B_C
  - for renaming function *r: W -> W* , *rename*(*r*) is a function from B_C to B_C

J. Burch, "Trace Algebra for Automatic Verification of Real-Time Concurrent Systems", *Ph.D. dissertation CMU-CS-92-179,* Carnegie Mellon University, Pittsburgh, PA, August 1992.

Trace Algebra

- A trace need not be a sequence. Any set for which "appropriate" projection and renaming functions are defined can be used as a trace
- The meaning of the operations of projection and renaming is defined by a set of axioms
  - Intuitively, the function *proj(B)*, for *B* a subset of *A*, takes a trace *x* and produces a trace *y* where the symbols not in *B* are dropped. This can be used to hide internal signals in the process of a composition
  - The function *rename(r)*, where *r: W -> W* is a bijection, renames the elements of a trace *x*. This function corresponds to the process of instantiation

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Trace Algebra

- **Axioms**
  - T1. *proj(B)(x)* is defined iff there exists an alphabet *A* such that $x \in A$ and $B \subseteq A$. When defined, *proj(B)(x)* is an element of $B_C(B)$
  - T2. *proj(B)(proj(B')(x))* = *proj(B)(x)*
  - T4. Let $x \in B_C(A)$ and $x' \in B_C(A')$ be such that $proj(A \cap A')(x) = proj(A \cap A')(x')$. For all *A''* where $A \cup A' \subseteq A''$ there exists $x'' \in B_C(A'')$ such that $x = proj(A)(x'')$ and $x' = proj(A')(x'')$.
  - T5. *Rename(r)(x)* is defined iff $x \in B_C(dom(r))$. When defined *rename(r)(x)* is an element of $B_C(codom(r))$.

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Example

- For every alphabet A over W, $B_C(A)$ is the set A^∞.
- $proj(B)(x)$ is the sequence formed from x by removing every symbol a not in B.
- $rename(r)(x)$ is the sequence formed from x by renaming every symbol a in x according to r.
- Must prove the axioms of trace algebra
- Traces are not necessarily sequences
  - Let $B_C(A) = 2^A$

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Trace Structures Algebra

- Let $C_C = (B_c, proj, rename)$ be a trace algebra over W. A trace structure T is a pair (A, P) where $P \subseteq B_C(A)$.
- Let TS be a subset of the trace structures. Then $A_C = (C_C , TS)$ is a trace structure algebra if TS is closed under *parallel composition, projection* and *renaming.*
- Parallel Composition:
  - $T_1 \,||\, T_2 = \{ x : proj(A_1)(x) \in T_1 \wedge proj(A_2)(x) \in T_2 \}$

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Conservative approximations

- Objective
  - To translate a problem in one domain into a similar (but more tractable problem) in another domain
  - Ensure that false positive do not occur
- Let $A_C = (C_C, TS)$ and $A'_C = (C'_C, TS')$ be trace structure algebras and consider two functions Ψ_u and Ψ_l from TS to TS'. We say that $\Psi = (\Psi_u, \Psi_l)$ is a conservative approximation if $\Psi_u(T_1) \subseteq \Psi_l(T_2)$ implies that $T_1 \subseteq T_2$.

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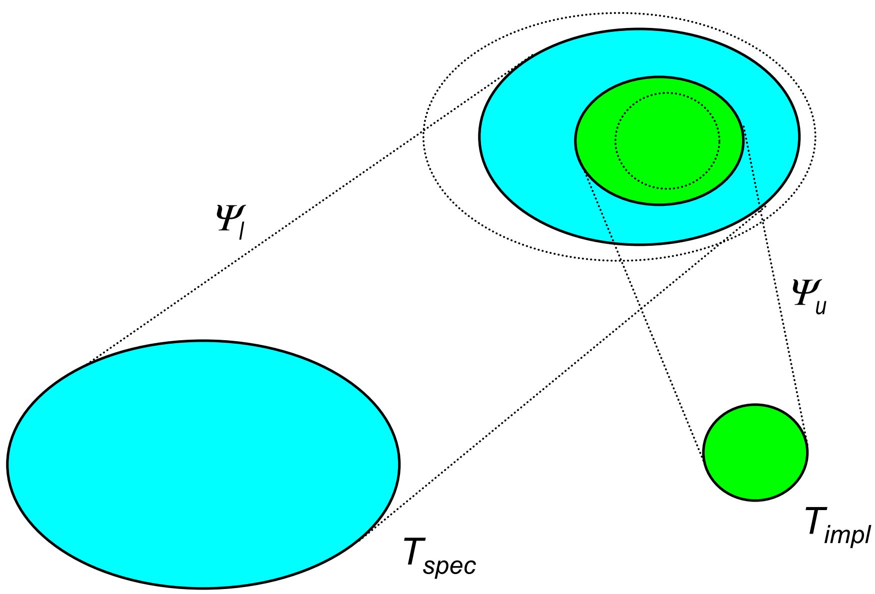
Conservative Approximations



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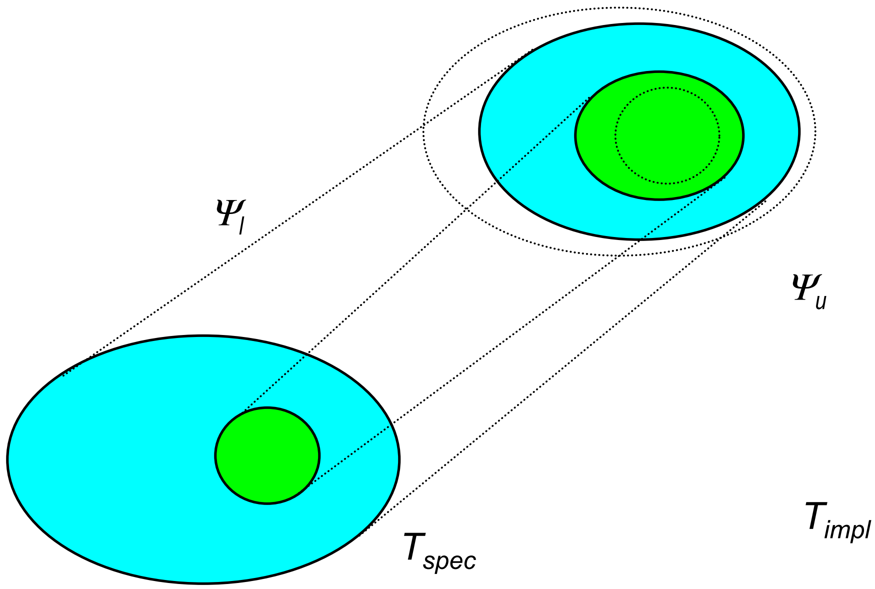
Conservative Approximations



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Homomorphisms on Trace Algebras

- Let C_C and C'_C be trace algebras. Let h be a function from B_C to B'_C such that if $x \in B_C(A)$ then $h(x) \in B'_C(A)$. The function h is a homomorphism iff
  - $h(proj(B)(x)) = proj(B)(h(x))$
  - $h(rename(r)(x)) = rename(r)(h(x))$
- Example: from A^∞ to 2^A
  - $h(x) = \{ a : \exists n [a = x(n)] \}$

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Approximations induced by homomorphisms

- If $x' = h(x)$ then intuitively x' is an abstraction of any trace y such that $h(y) = x'$. Then x' represents all such y.
- Then define $\Psi = (\Psi_u, \Psi_l)$ as follows
  - $\Psi_u(T) = h(T)$
  - $\Psi_l(T) = h(T) - h(B_C(A) - T)$
- Note that intuitively $\Psi_u^{-1}(T') \supseteq T \supseteq \Psi_l^{-1}(T')$.
- Theorem: Ψ is a conservative approximation
- If $\Psi_u^{-1}(T') = T = \Psi_l^{-1}(T')$ then we can define an inverse Ψ^{-1}

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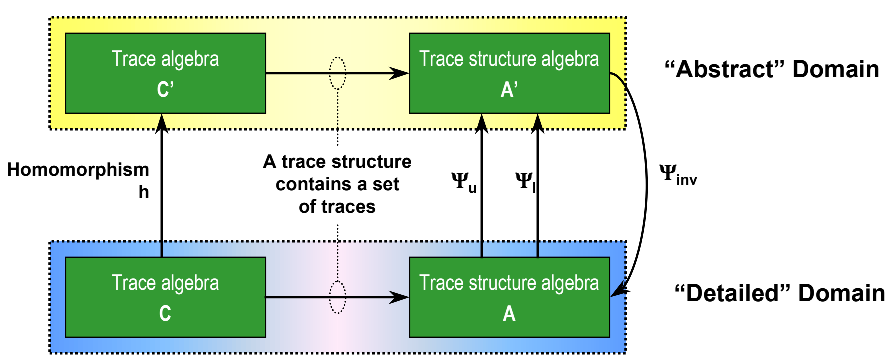
Trace algebras and Trace Structures algebras



Let T_{spec} and T_{impl} be trace structures in A. Then

if $\Psi_u(T_{impl}) \subseteq \Psi_l(T_{spec})$ then $T_{impl} \subseteq T_{spec}$

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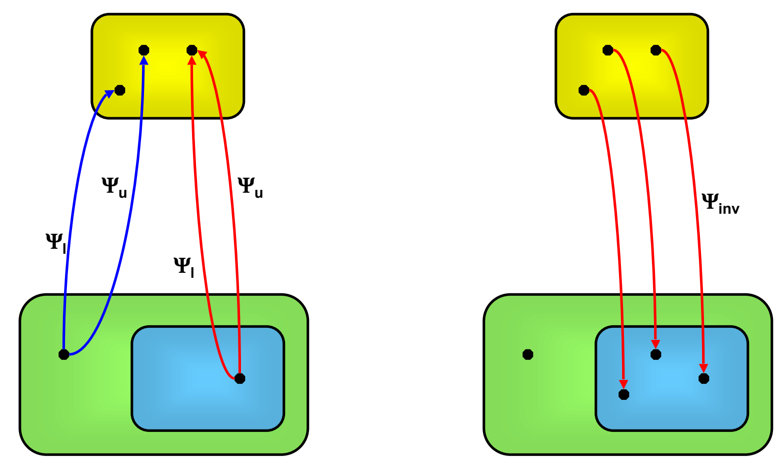
Inverses of Conservative Approximations



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Parallel composition

- Example based on the theory of trace structures
- Parallel composition defined in each domain in terms of a projection operation
  - Data flow $\quad f : A \rightarrow V^* \quad \text{proj}(B)(f) = f\,|_B$
  - Synchronous $\quad \langle g \rangle \in (A \rightarrow V)^* \quad \text{proj}(B)(\langle g \rangle) = \langle g\,|_B \rangle$
  - Discrete $\quad N \rightarrow (A \rightarrow V) \quad \text{proj}(B)(N \rightarrow g) = N \rightarrow g\,|_B$
- For each domain
  - $T_1 \,||\, T_2 = \{ x : \text{proj}(A_1)(x) \in T_1 \wedge \text{proj}(A_2)(x) \in T_2 \}$

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Conclusions

- We are defining semantics domains for the representation of various models of computation
- Using the formal techniques to study heterogeneous interaction
- Work in progress in generalizing the foundation layer to cover representation of different aspects

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Basic elements: a model

- A model is a representation of an entity (an object or an idea)
- In the previous example we use mathematical structures as representations
- In our approach we use the theory of a structure as a logical representation
  - Theory of a structure: the set of true statements about the structure in a logic
- A model is a set of properties that must be satisfied by the represented object
  - Neutral with respect to representation

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Classes of models and local refinement

- A class of models is represented by the set of properties Φ common to all models
- A model Ψ belongs to a class Φ if and only if $\Psi \models \Phi$
  - This notion corresponds to local refinement
  - Ψ must have all the properties of Φ plus (possibly) some more
- For example if Φ is the class of models with a partial order, then a total order Ψ belongs to the class Φ

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Inter-class Refinement

- Let P and Q be two classes of models
- Define when elements $p \in P$ and $q \in Q$ represent the same underlying object
- Bipartite equivalence (or correspondence)
  - Let Φ_A be a set of assertions that defines a notion of correspondence. We say that p and q are bipartite equivalent if and only if their disjoint union satisfies Φ_A.
- The theory Φ_A outlines what must be true in order for two heterogeneous models to represent the same entity

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Example: abstraction of time

- Abstract time away. For all properties φ

  Φ_A: $\forall x\,(\,\varphi(\,x\,) \leftrightarrow \forall t\ \varphi(\,f(\,x, t\,)\,)\,)$

  Φ_A': $\forall x\,(\,\varphi(\,x\,) \leftrightarrow \exists t\ \varphi(\,f(\,x, t\,)\,)\,)$

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Abstraction or refinement?

- q is a refinement of p if q knows everything about p
  - As in local refinement, we want q $\models$ p
- But in order to do that we need the information on the bipartite equivalence
  - $(\,q \cup \Phi_A\,) \models p$
- Can be extended to classes of models
  - $(\,\Phi_Q \cup \Phi_A\,) \models \Phi_P$
- Transitive and reflexive relation: a pre-order

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Constraints

- A constraint is a property that must hold, but that is derived from a corresponding model at a different level of abstraction
- Let P, Q be classes of models identified by specifications Φ_P and Φ_Q. Let Φ_A be a theory of equivalence.
- Theorem: If $p \in P : p \not\models (\Phi_Q \cup \Phi_A)^{\models}$, then there is no model $q \in Q$ such that p and q are equivalent
- Hence we define the constraints of Q over P, mediated by equivalence Φ_A, as the consequence closure
  - $\Psi = (\Phi_Q \cup \Phi_A)^{\models}$

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Interaction as a constraint application

- A special case of constraint application
- A theory Φ_C that defines how a set of properties is translated into another set of properties
- Essential for heterogeneous systems (no notion of parallel composition)
- But also useful for homogeneous systems
  - synchronous vs. asynchronous automata composition

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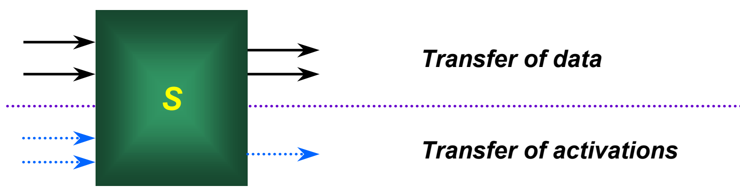
Assume-Guarantee for executable models

- For a class of executable models, declare the local model of computation:
  - For each model, declare the properties of data communication
  - For each model, declare the properties of activations
- The less stringent the properties, the more reusable the component



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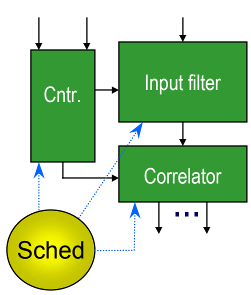
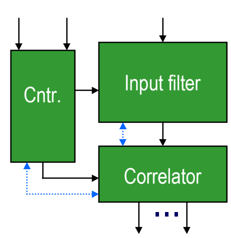
Constructing networks

- Properties are divided into requirements (assumptions) and guarantees

An explicit scheduler (or director) can be used to embed the resolution of assumption and guarantees



A collection of components must mutually satisfy their requirements through the use of guarantees



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Conclusions

- By providing a common formal framework for heterogeneous systems we address
  - The problem of maintaining consistency through the refinement process and the design flow
  - A way to consciously reason about the interaction between heterogeneous models
- This provides a more precise verification that lowers time to market and increases productivity

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Conjoint structures

- A way to talk about two structures at the same time
- Given structures *A* and *B*, in languages *S* and *S*', consider the disjoint union *U* in language *S*"
- In general it is not the case that if $A \models \varphi$ then $U \models \varphi$
- However, let φ' be φ where all variables are constrained to range over the domain of *A*
- Theorem: $A \models \varphi$ if and only if $U \models \varphi'$
  - By induction and by the definition of truth

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A special case: Executable models

- Executable models are those that can be run as a simulation
- Characterized by a semantics of internal execution and of interaction
- The interaction is composed of
  - the way the data is exchanged
  - the relationships between the data transfers and the activations

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Example of requirements

- Data flow: define a partial order
  - For each activation, a sufficient amount of data must been seen at the inputs in the past
- Synchronous: define a total order
  - For each activation, corresponding data must be seen at the same time at the inputs
- Synchronous guarantees satisfy data flow requirements
- Sub-type (refinement) when $R \Rightarrow R'$ and $G' \Rightarrow G$
- Contravariant as in type systems

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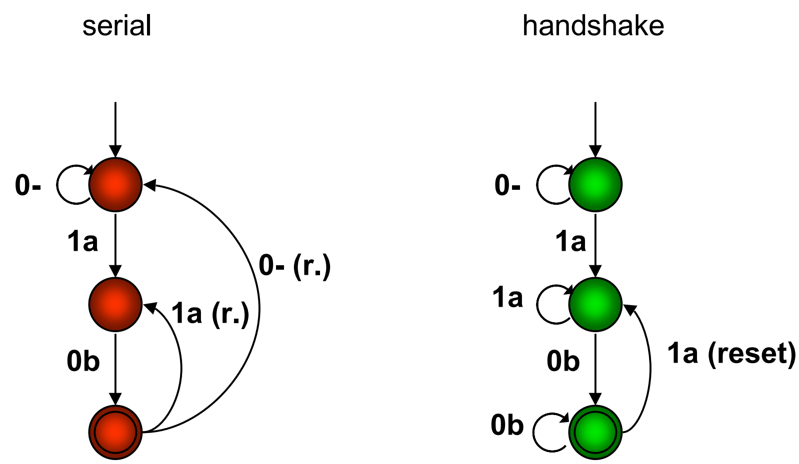
Representation of properties



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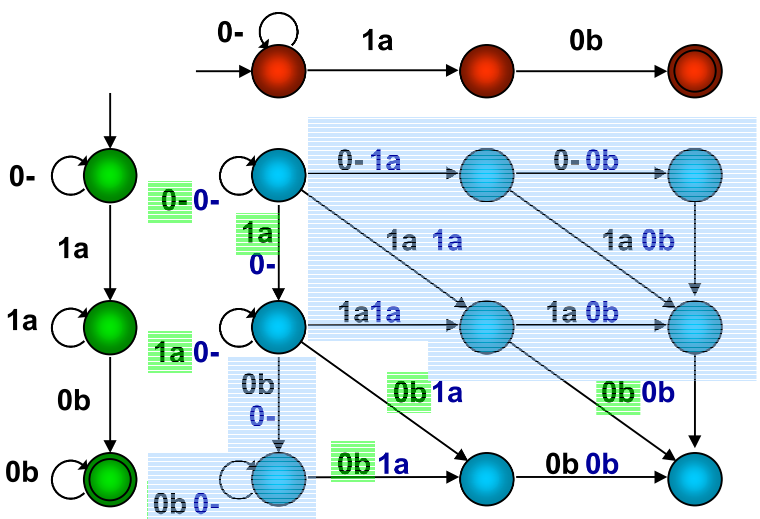
Derivation of requirements



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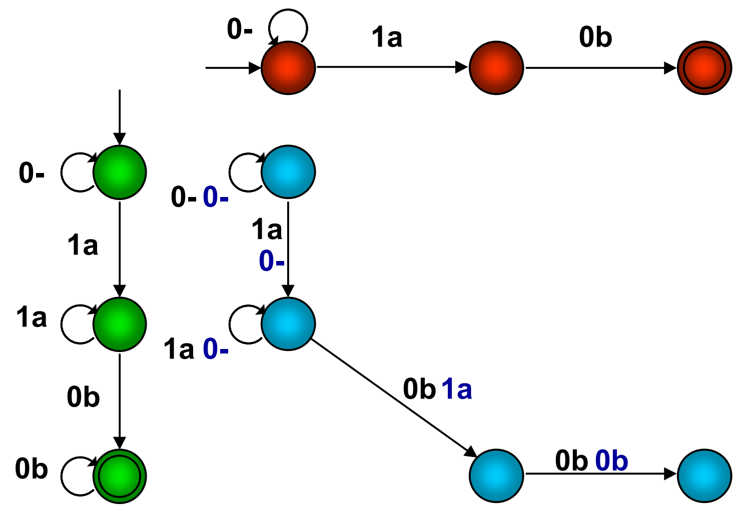
Derivation of requirements



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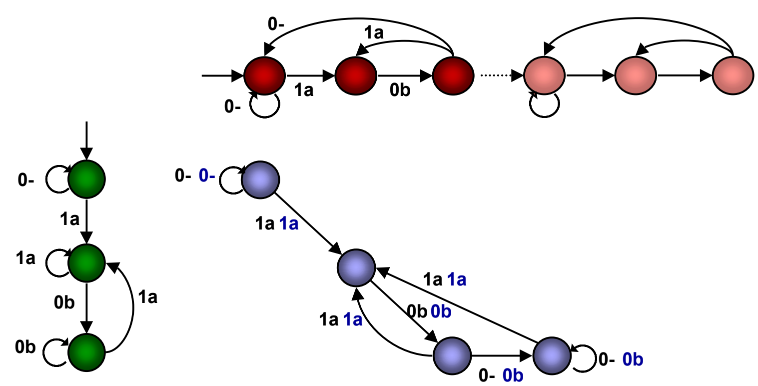
The satisfaction case



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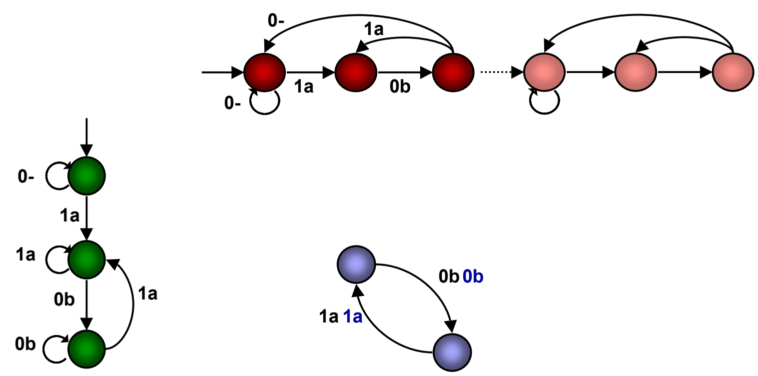
The satisfaction case



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Choosing the model

- Choosing the appropriate model is essential because
  - If the model is too detailed:
    - over-specification (miss opportunity for optimization)
    - complexity (must deal with too many details)
    - difficult to analyze (may not be possible to extract relevant properties)
  - If the model is too abstract:
    - under-specification
    - unwanted non-determinism
- ⇒ Can't really just do with one!
  - But a common infrastructure is necessary

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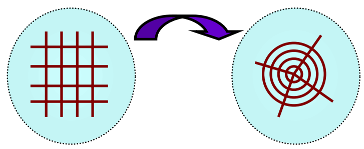
Domain transformation



Convert a representation from one model to another while preserving the properties of interest

- **Classes of transformations**
  - Property preserving transformation (homomorphism)
  - Identity preserving transformation (injective)
  - Abstracting transformation (strictly non-injective)
  - Non-deterministic transformation (relation: one-to-many)
  - Sub-typing transformation (embedding)

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