Heterogeneous Modeling: Hybrid Systems

- Hybrid Models
  - Automotive Powertrain
- Languages and Verification Problems
  - Simulink and StateFlow
  - CheckMate
  - Charon
  - Masaccio

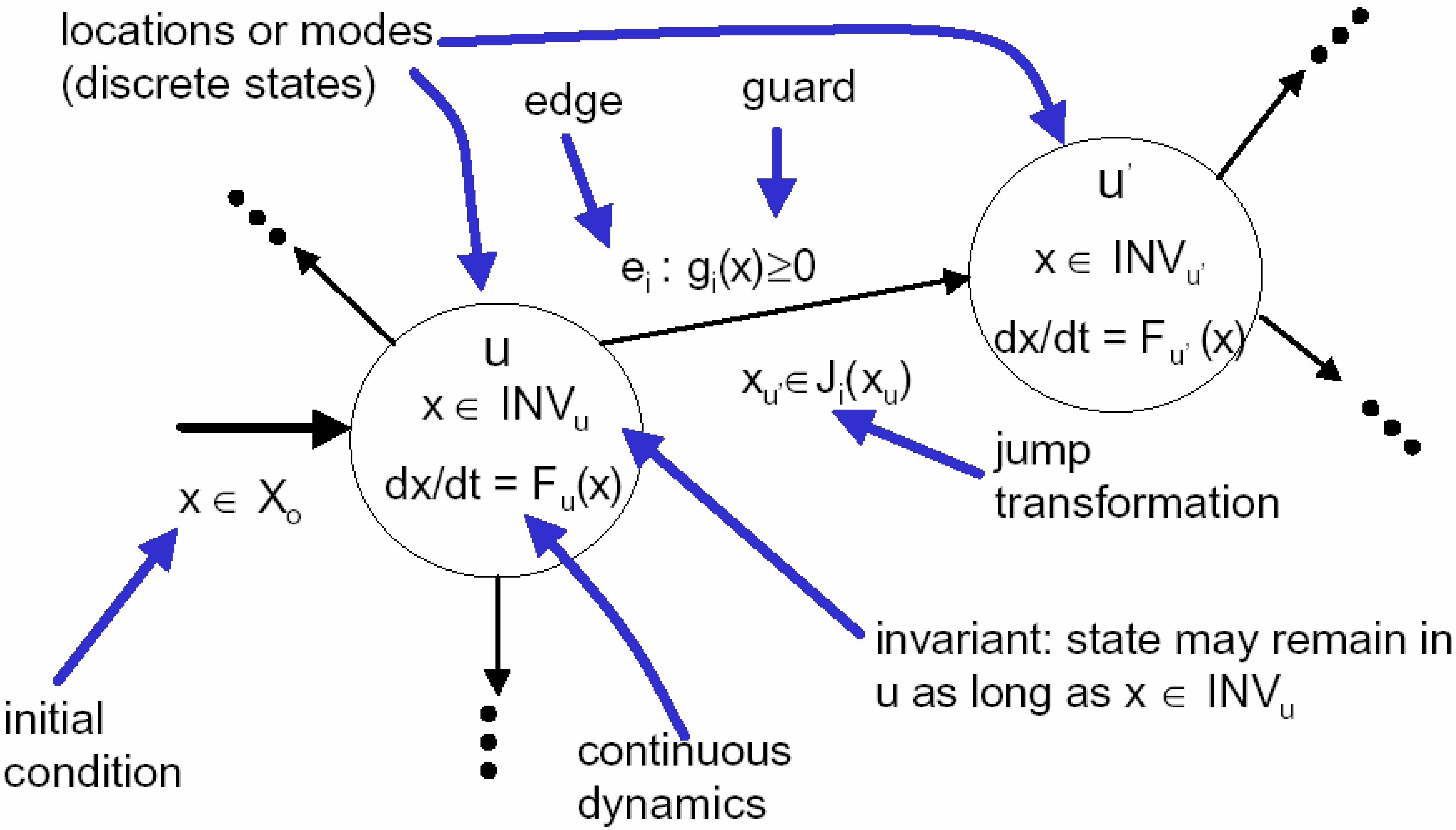
Motivation

- Hybrid Systems are becoming a major modeling paradigm for embedded systems
  - Capability of modeling controller and plant
  - Use of concurrent multiple levels of abstraction
- Difficult to verify and design
  - Combination of continuous and discrete dynamics of different types
  - Lack of "operationally strong" theoretical results
- Variety of tools and approaches mutually incompatible due to modeling differences

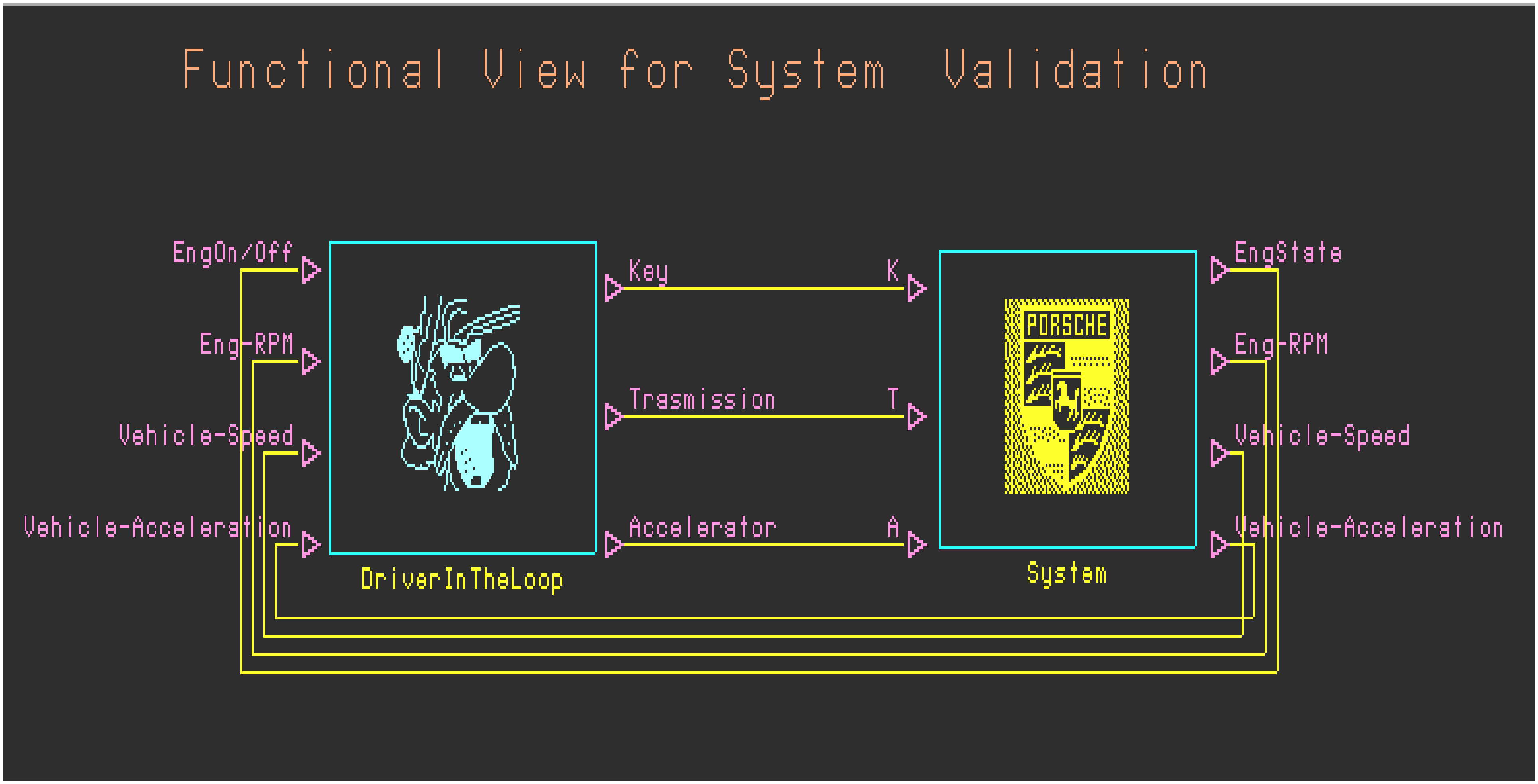
Foundations of Hybrid Model

- Used classic model by J. Lygeros, S. Sastry and C. Tomlin as basis
- Model consists of three parts:
  - Structure= sets, discrete and dynamical components
  - Time Bases= intervals over which behavior is continuous
  - Hybrid execution= rules according to which we have jumps and continuous flows
- Observations:
  - Non deterministic behavior allowed (needed)
  - Fixed interaction structure

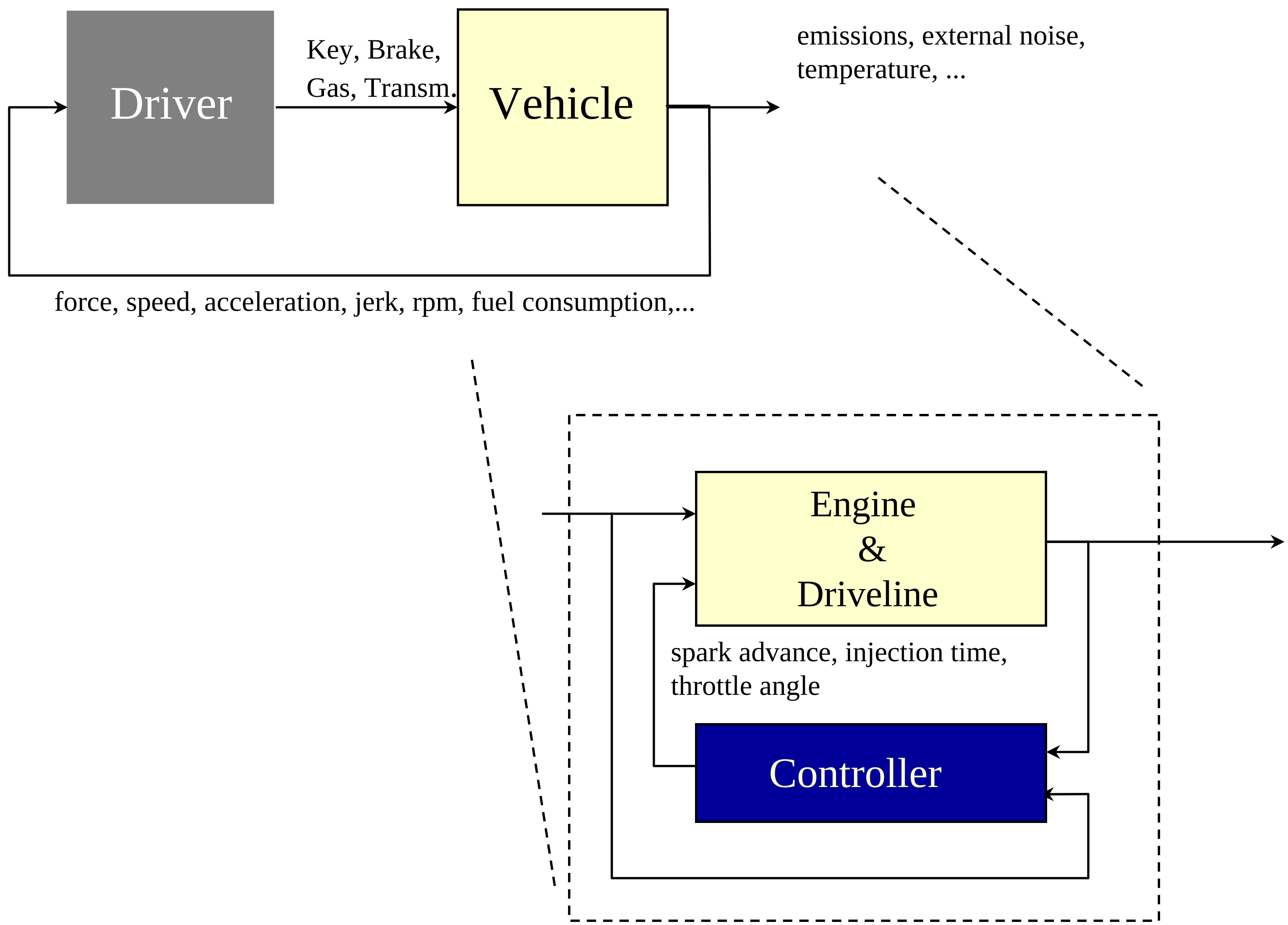
Model 1: Hybrid Automata

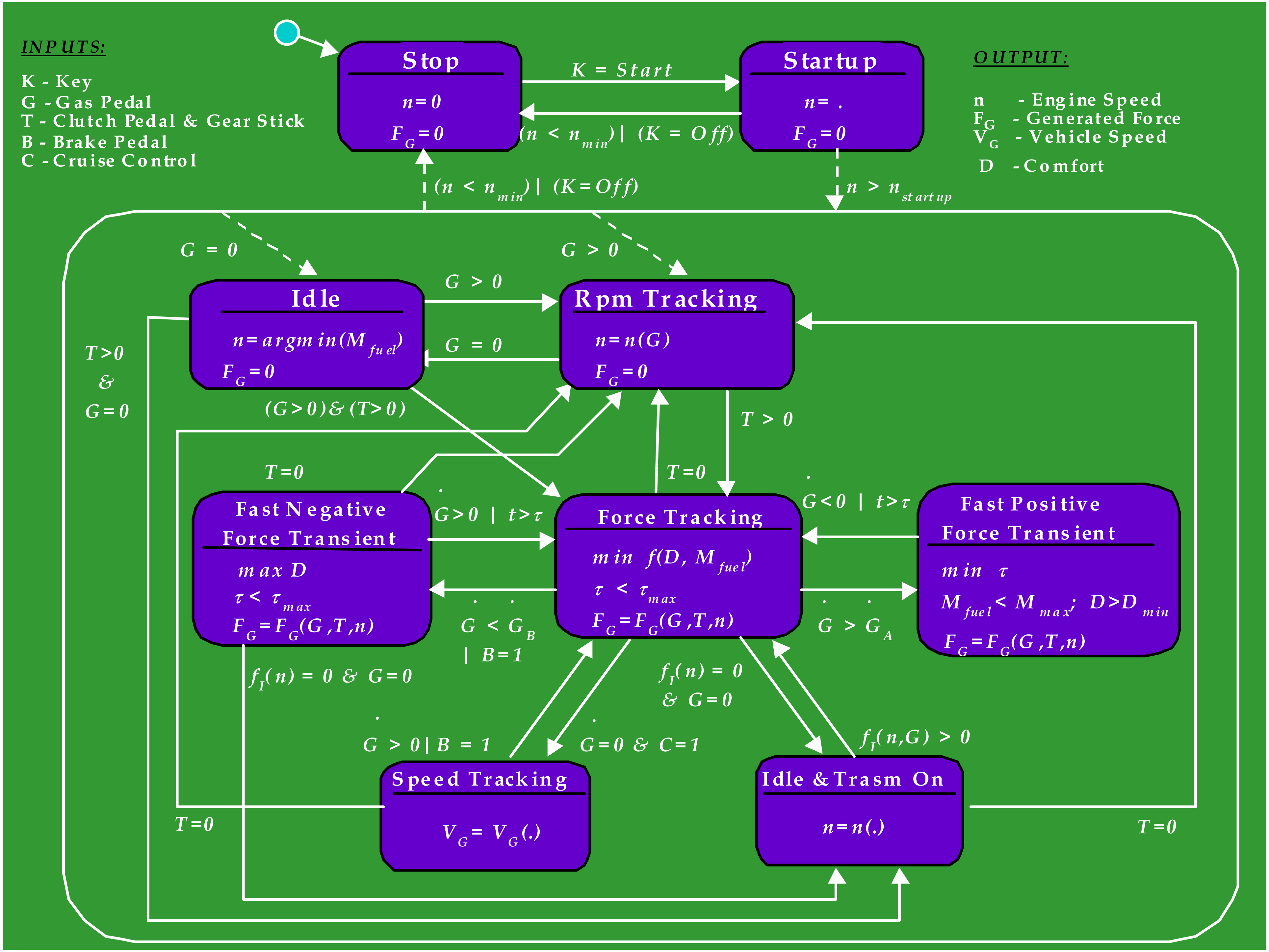


System Specifications



Closed loop vehicle model





INPUTS:
K - Key
G - Gas Pedal
T - Clutch Pedal & Gear Stick
B - Brake Pedal
C - Cruise Control
Stop
n=0
F_G=0
K = Start
(n < n_min) | (K = Off)
Startup
n= .
F_G=0
OUTPUT:
n - Engine Speed
F_G - Generated Force
V_G - Vehicle Speed
D - Comfort
(n < n_min) | (K=Off)
n > n_startup
G = 0
G > 0
Idle
n=argmin(M_fuel)
F_G=0
G > 0
G = 0
Rpm Tracking
n=n(G)
F_G=0
T>0 & G=0
(G>0)&(T>0)
T > 0
T=0
T=0
Fast Negative Force Transient
max D
τ < τ_max
F_G=F_G(G,T,n)
G>0 | t>τ
Force Tracking
min f(D, M_fuel)
τ < τ_max
F_G=F_G(G,T,n)
G<0 | t>τ
Fast Positive Force Transient
min τ
M_fuel < M_max; D>D_min
F_G=F_G(G,T,n)
G < G_B | B=1
G > G_A
f_I(n) = 0 & G=0
f_I(n) = 0 & G=0
G > 0 | B = 1
G=0 & C=1
f_I(n,G) > 0
Speed Tracking
V_G= V_G(.)
Idle & Trasm On
n=n(.)
T=0
T=0

Why Mixed Models of Computation in internal combustion engines control ?

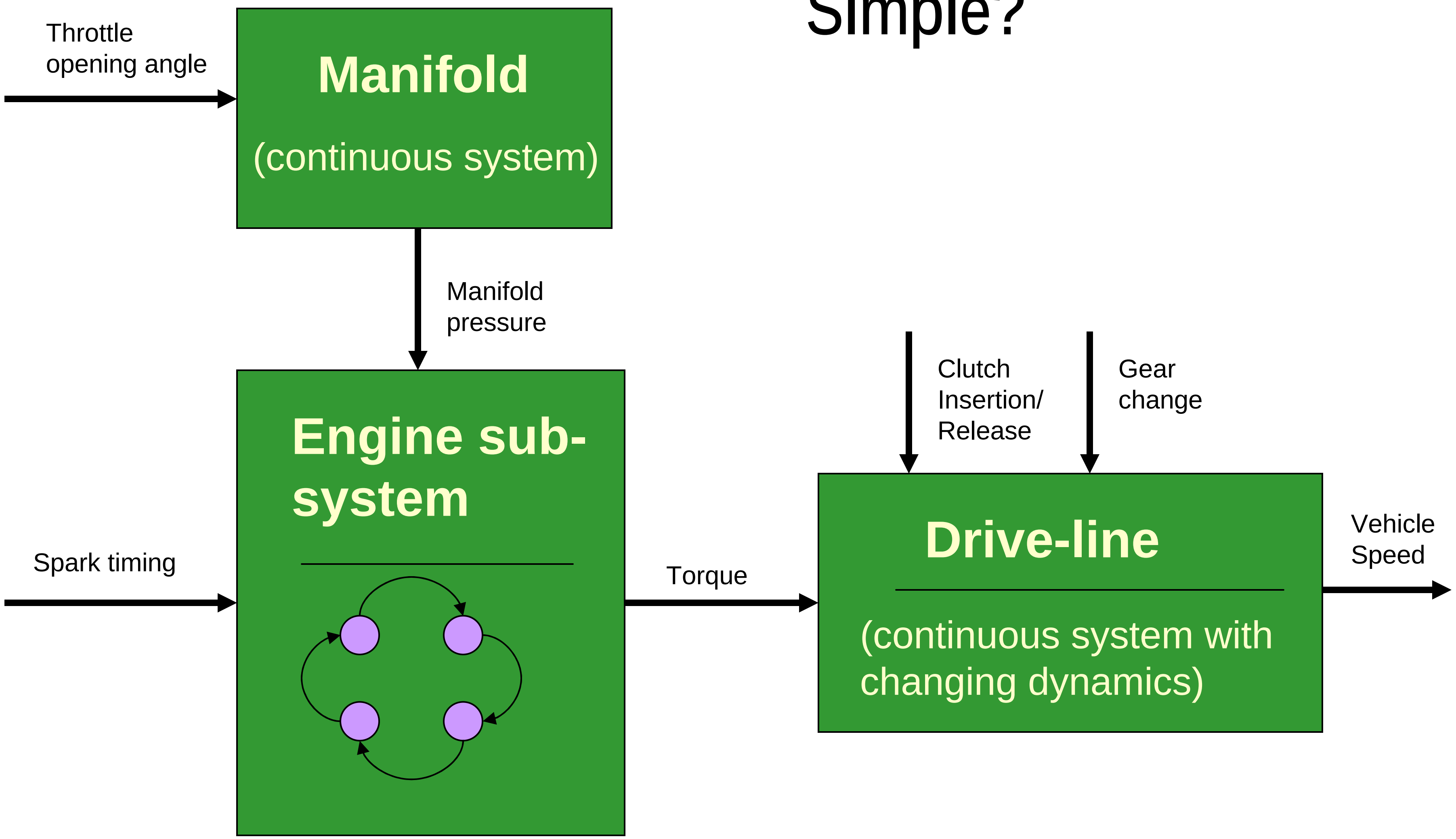
- Specifications
- Control variables

input	value	time
Throttle valve	Continuous	Continuous
Fuel injection	Continuous	Discrete
Spark ignition	Discrete	Discrete

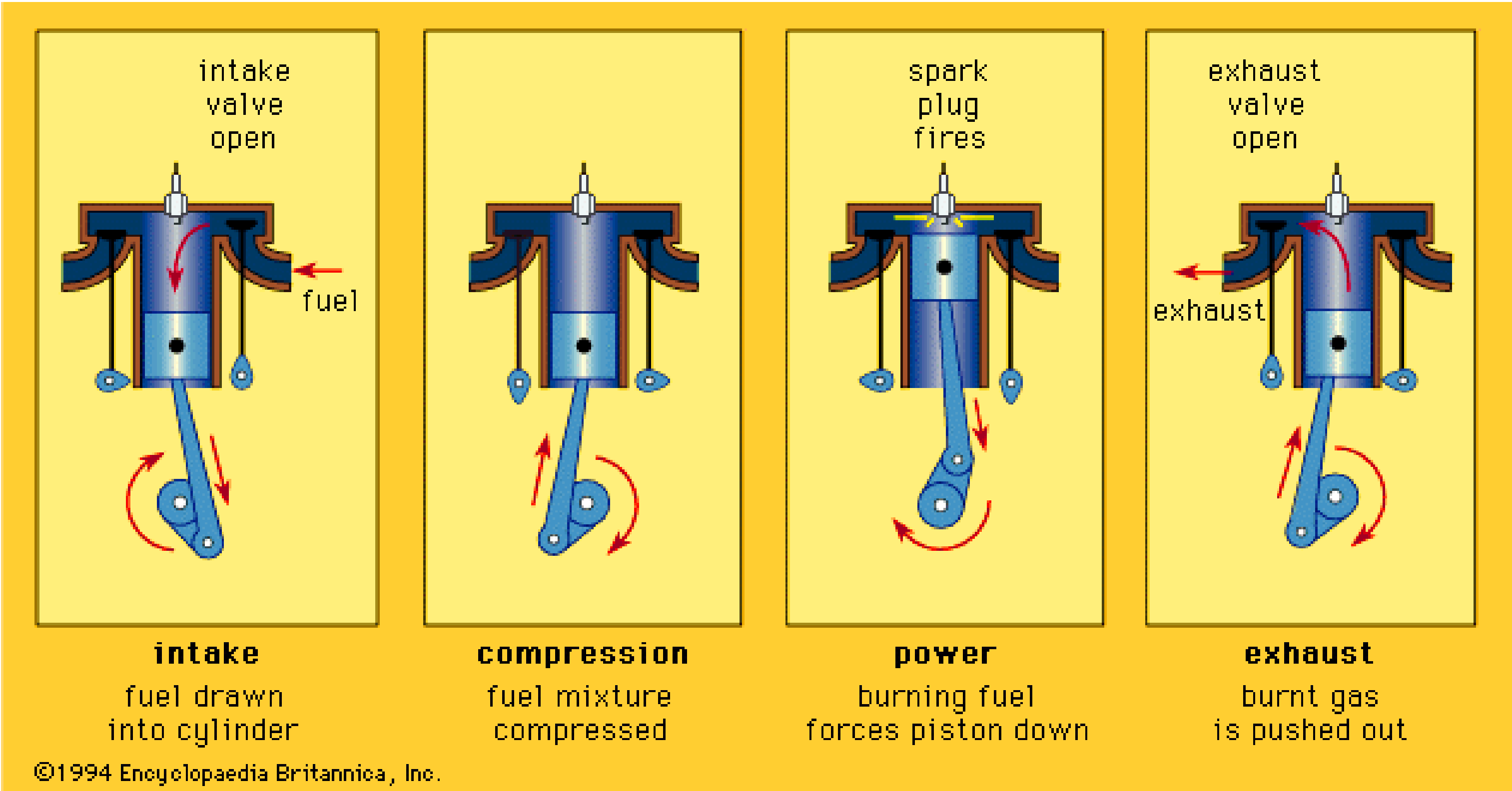
- Physical processes in the plant
- Description of the HW/SW implementation constraints

Model of Power-train

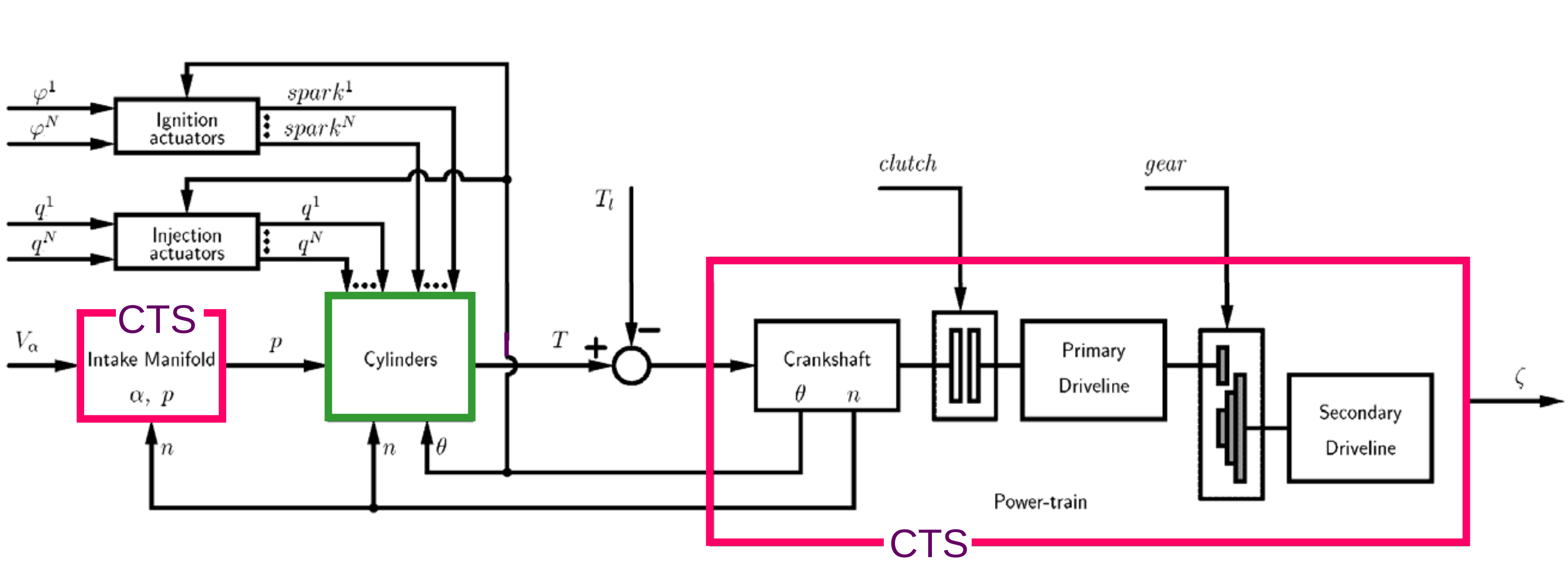
Simple?



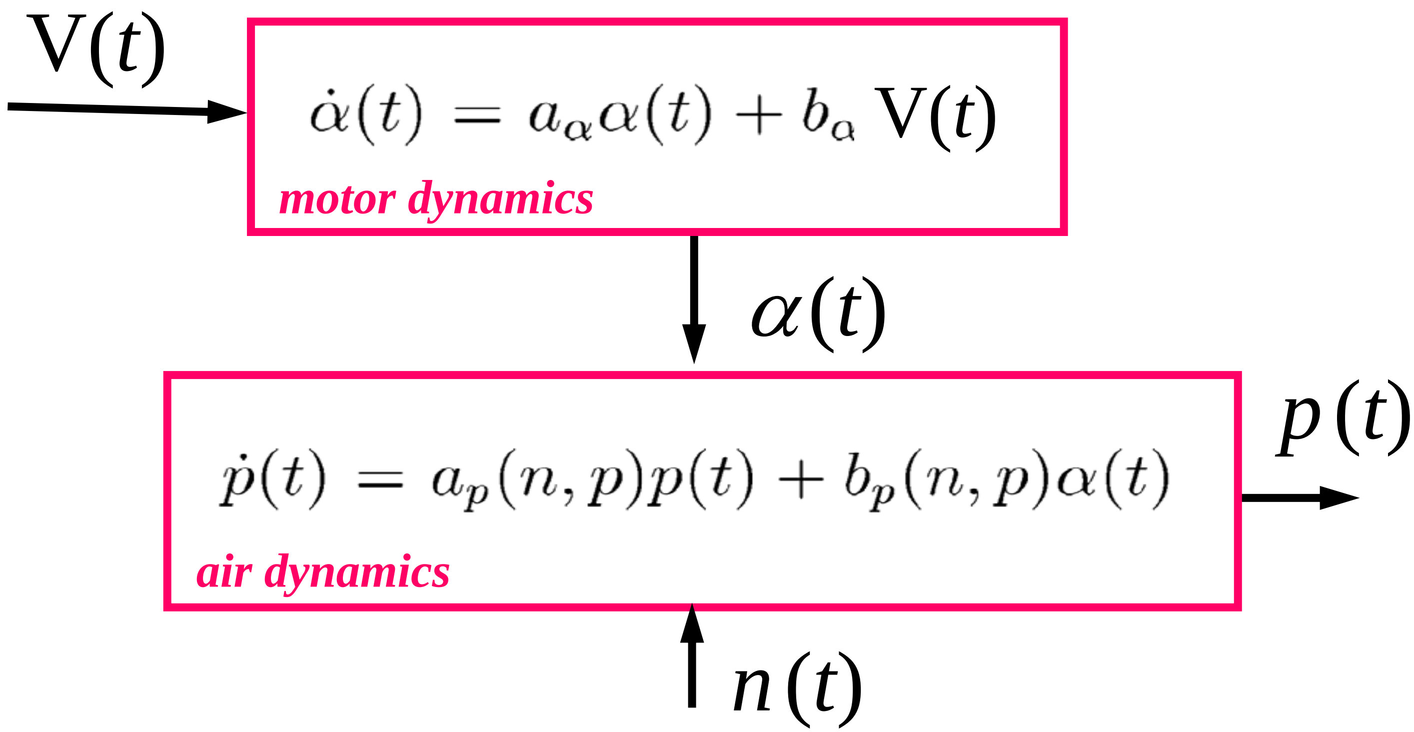
4-stroke engines



Engine and Power-train Model



Intake Manifold



$V(t)$ - *throttle motor voltage*

$\alpha(t)$ - *throttle angle*

$n(t)$ - *crankshaft speed*

$p(t)$ - *manifold pressure*

Power-train

$T(t)$

$$\begin{bmatrix} \dot{\alpha}_e(t) \\ \dot{n}(t) \\ \dot{\omega}_p(t) \end{bmatrix} = A \begin{bmatrix} \alpha_e(t) \\ n(t) \\ \omega_p(t) \end{bmatrix} + b\,T(t) + b_0$$

$$\dot{\theta}(t) \quad = 6\,n(t)$$

$\zeta(t)$

power-train dynamics

$\zeta(t)$

$\theta(t)$

***T (t)** - generated torque*

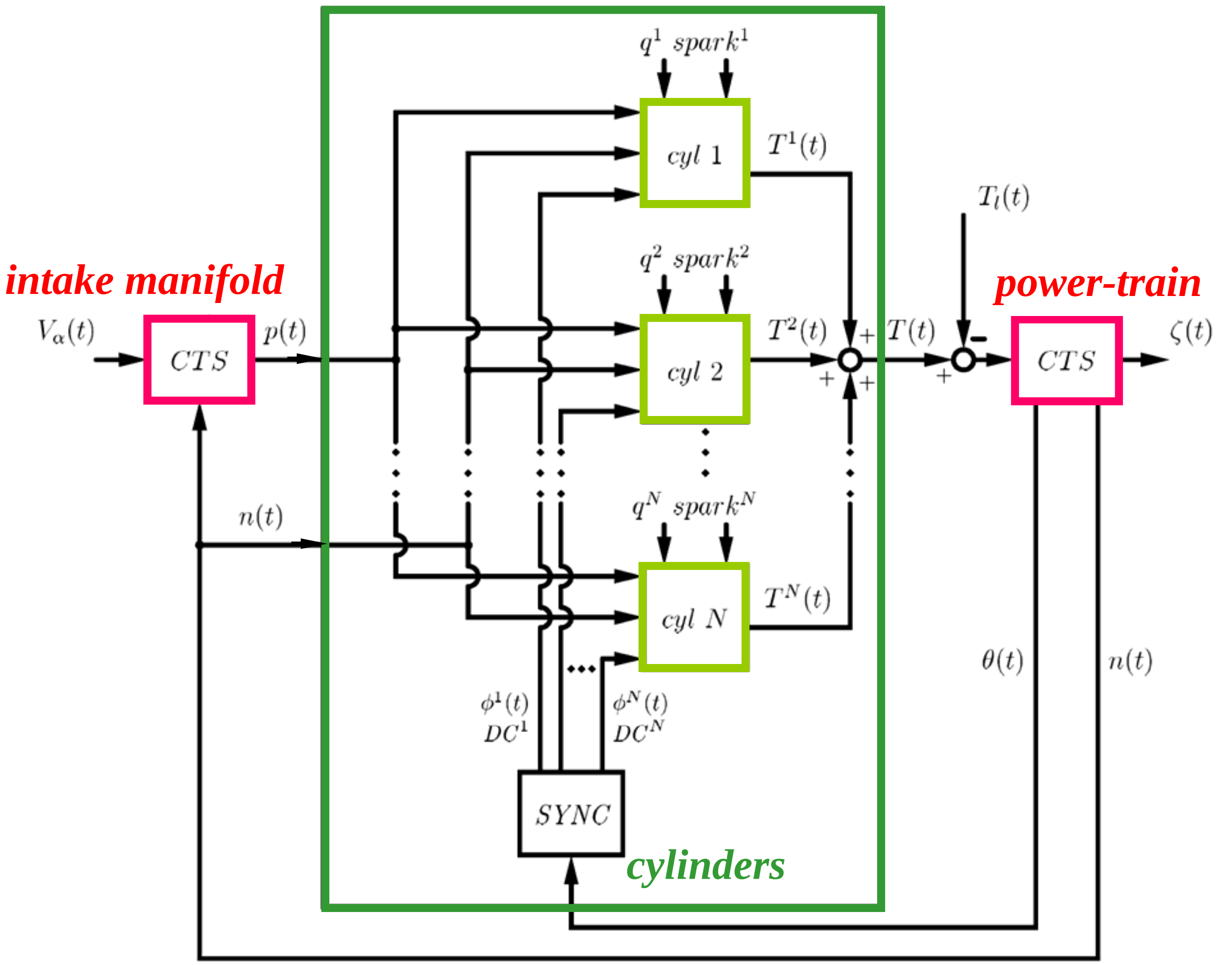
***θ (t)** - crankshaft angle*

***α_e(t)** - torsion angle*

***n (t)** - crankshaft speed*

***ω_p (t)** - wheel speed*

Engine and Power-train Model



Hybrid Systems in the Tagged-Signal Models (TSMs) Framework

Hybrid systems can be seen *as formalisms* for describing a complex system using mixed models of computation when a single one is not powerful, expressive or practical enough.

- An event $e \in V \times T$: V is the set of values and T is the set of tags e.g.
  - universal time (T is the set of real numbers)
  - discrete time (T is a totally ordered discrete set),
- a signal is a set of events,
- a process with N channels is a subset of the set of N-tuples of signals.

Models of Computation

- A *Finite State Machine (FSM)* is a synchronous TSM process in which the tags take values in N and the inputs, outputs and states take values on finite sets.

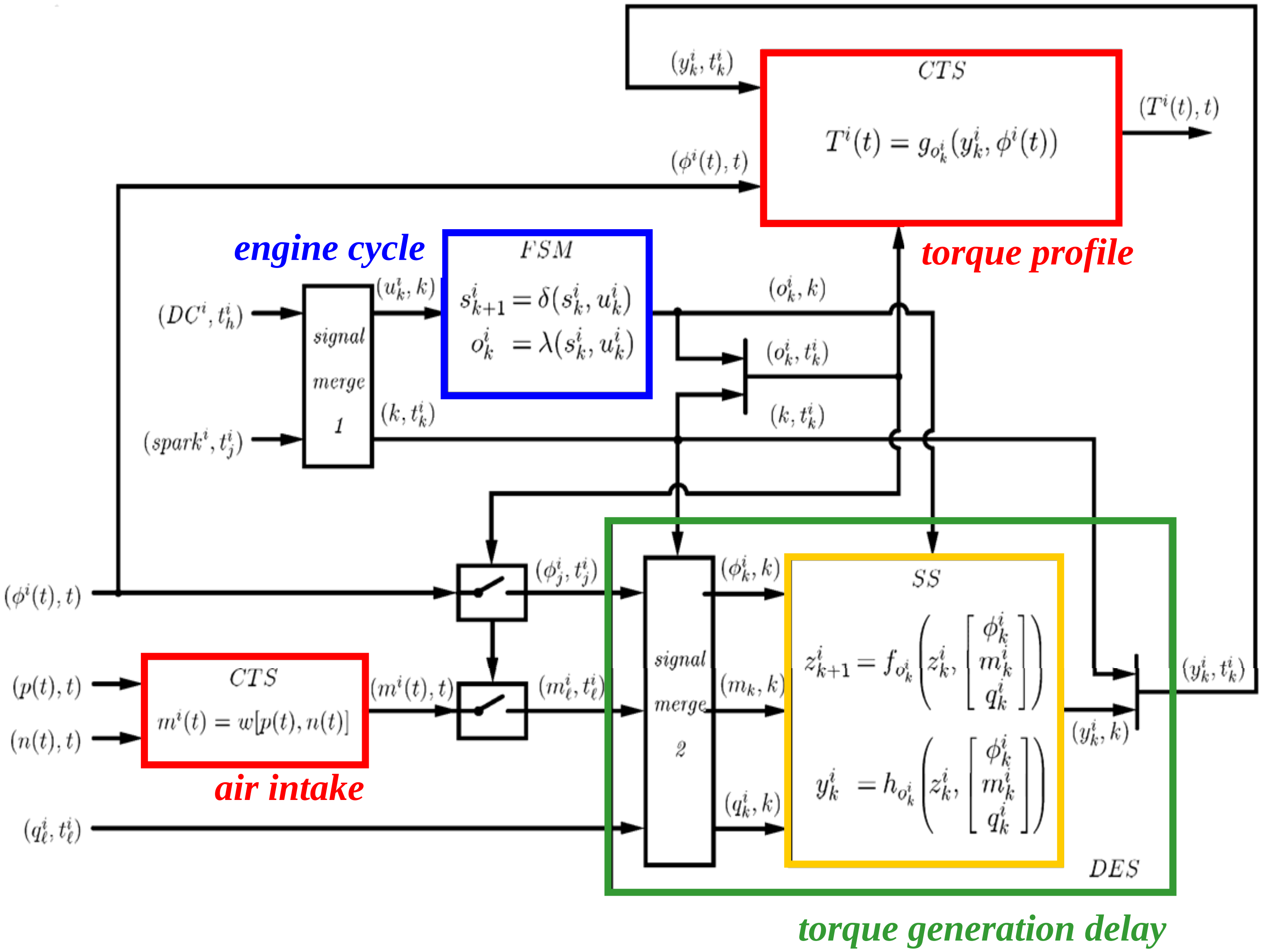
- A *Sequential System (SS)* is a synchronous TSM process in which the tags take values in N and the inputs, outputs and states assume values on infinite sets.

- A *Discrete-Event System (DES)* is a timed TSM process in which the tags are order-isomorphic with N (and denote instants of time).

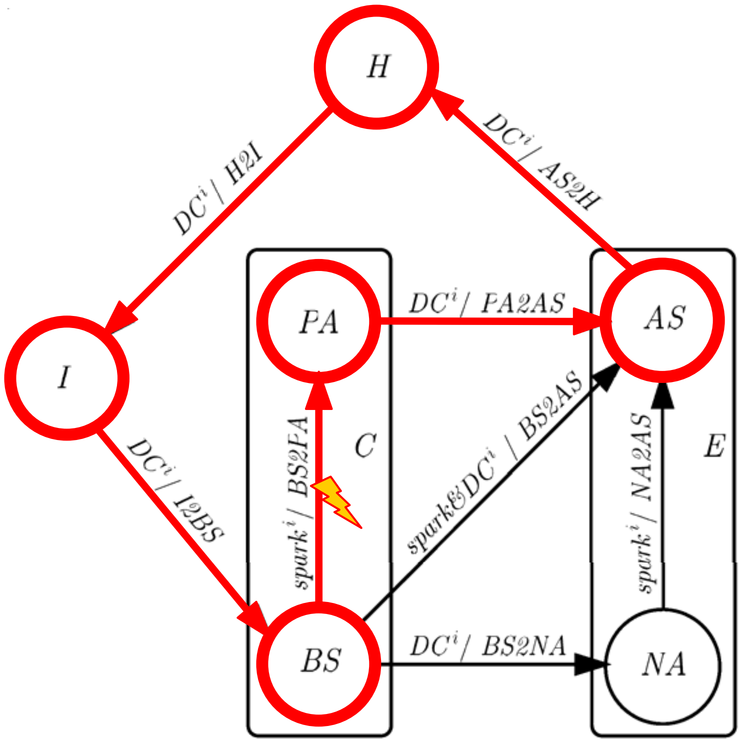
- A *Continuous-Time System (CTS)* is a timed TSM process in which the tags take values in a connected set on which a metric is defined (and denote instants of time).

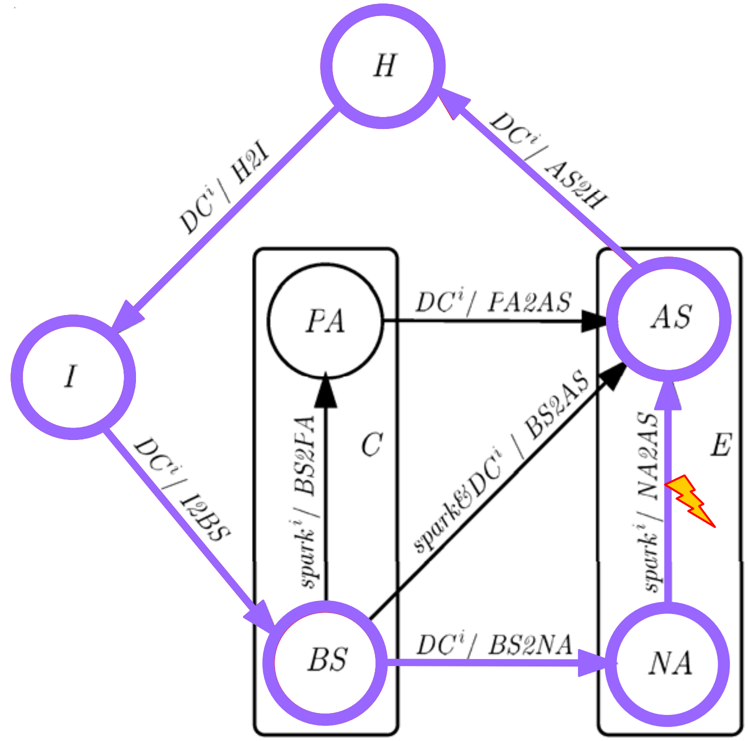
- A *Discrete-Time System (DTS)* is a synchronous DES.

Hybrid Tagged-Signal Model of a Single Cylinder



Engine Cycle (FSM)





- **positive spark advance:**

the spark is given before the TDC between the compression and expansion strokes.

- **negative spark advance:**

the spark is given after the TDC between the compression and expansion strokes.

Torque Generation Delay (SS)

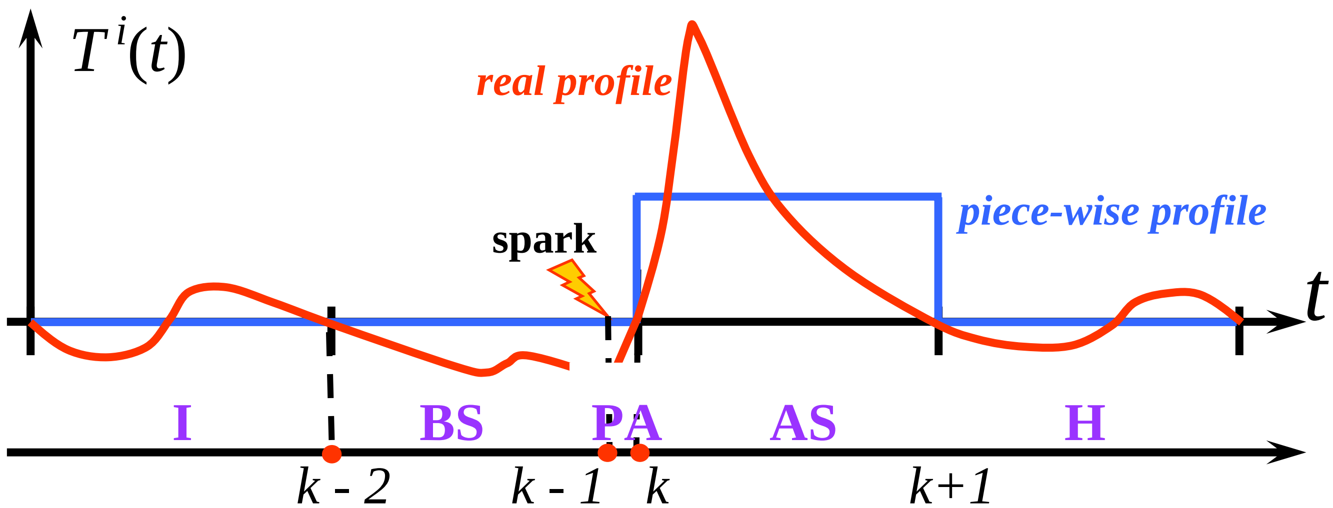
Sequential System

$$y_k^i = (m^i, q^i, \varphi^i) = (m_{k-2}^i, q_{k-2}^i, 180^o - \phi^i(t_{k-1}))$$

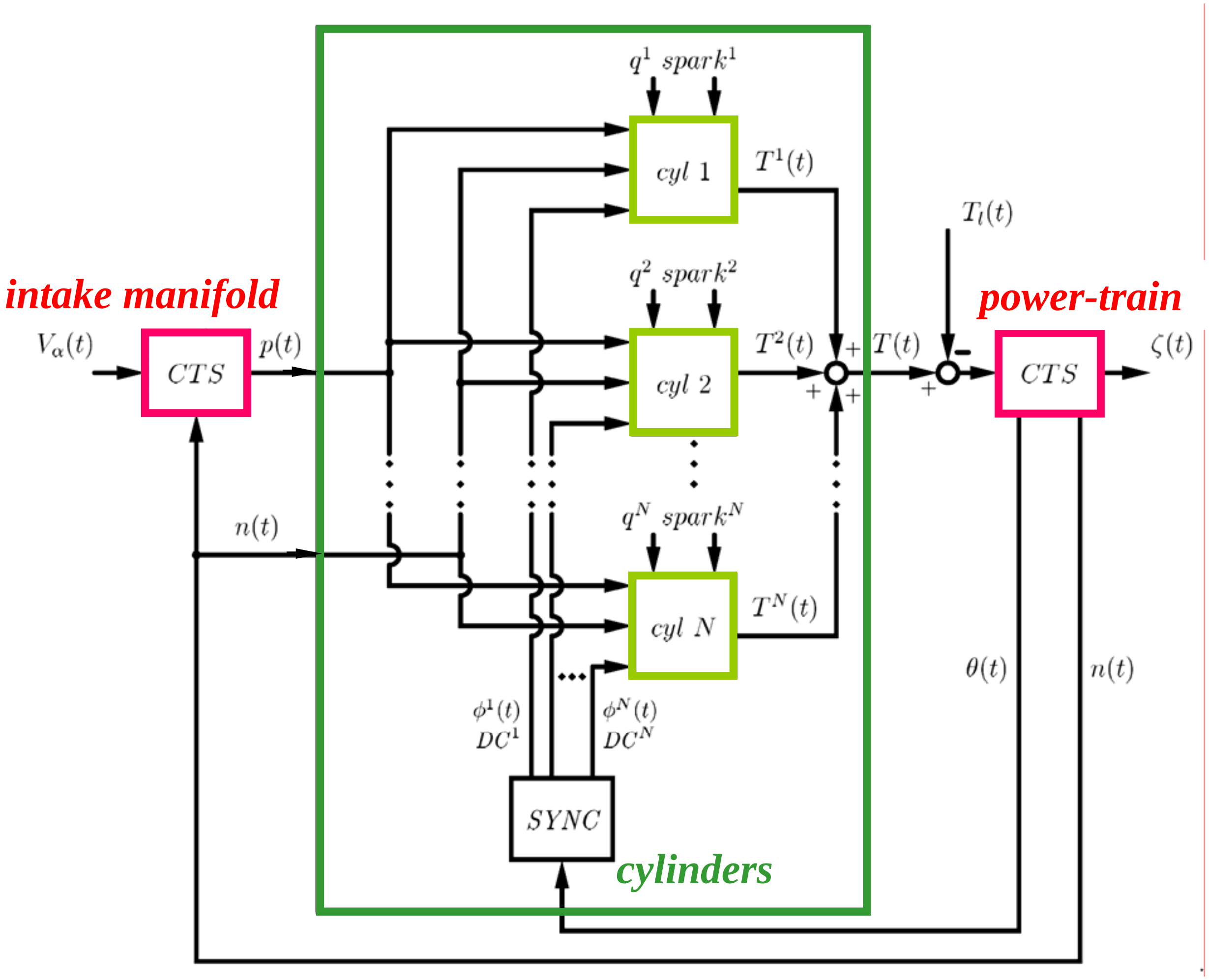
Torque Profile (CTS)

Continuous-Time System

$$T^i(t) = g_{o_k^i}(y_k^i, \phi^i(t))$$



Engine and Power-train Model



Hybrid Model vs Mean-Value Model

- **Mean-Value Model: accurate over a longer time window**
  - regulation control problems
  - low performance transient problems
- **Hybrid Model: cycle accurate**
  - transient control problems
  - stability of delay-sensitive control algorithms
  - high performance control algorithms

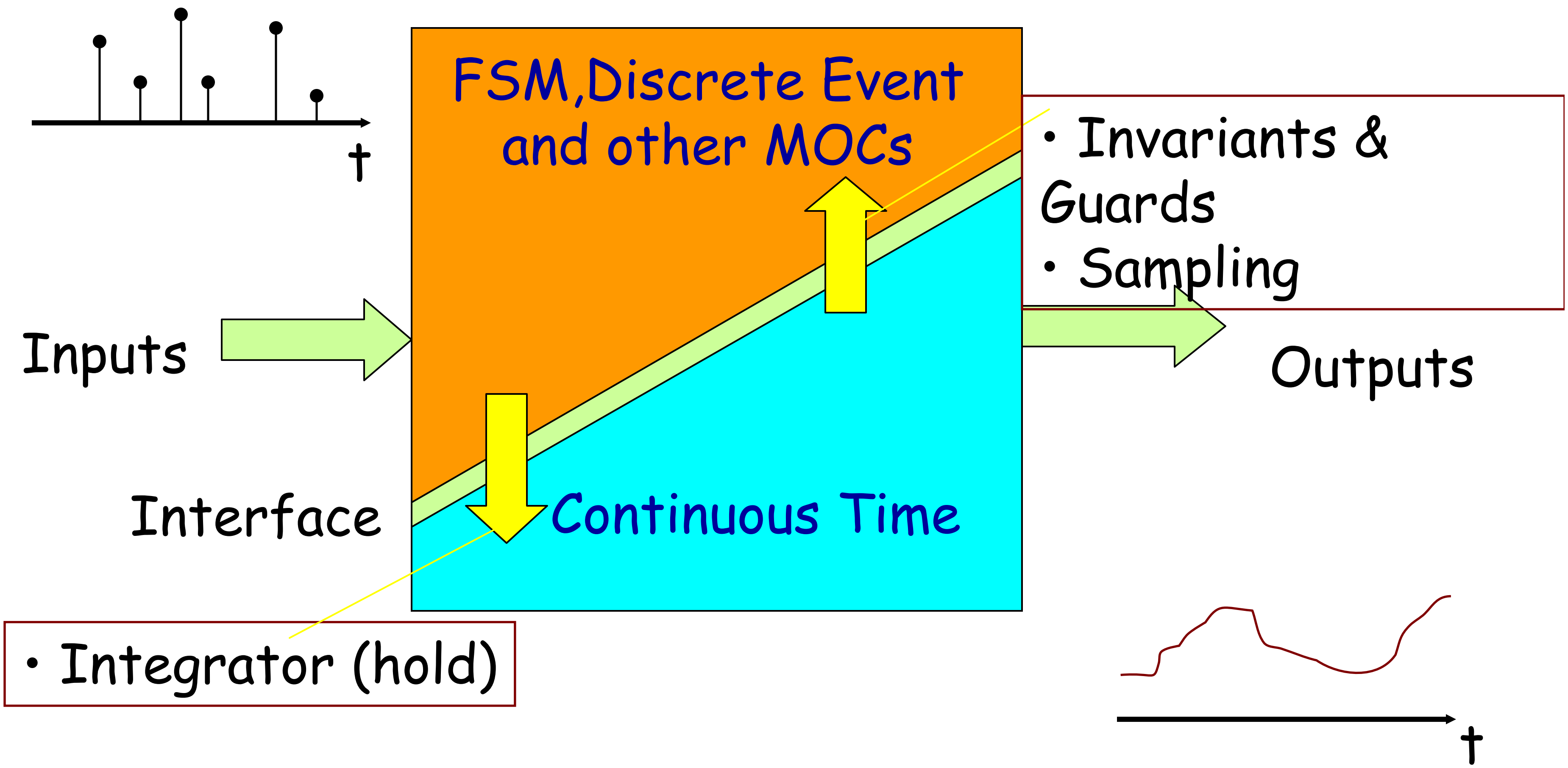
Outline

- Hybrid Models
- Languages and Verification Problems
  - Simulink and StateFlow
  - CheckMate
  - Charon
  - Masaccio

What is a simulator?

- Given a mathematical model of the system, computes its evolution and its outputs under a pre-determined set of inputs
- The mathematical model expresses heterogeneity and concurrency
- The simulator computes the response of the model by mapping it onto the "device" used to carry out the computation
- In general, the computing device has limited resources and is digital
  - We must embed the model of time of the model into the model of the computing device that gives the "common denominator" (e.g., discretize time, synchronize)
  - We must map a set of concurrent processes into a sequential system (e.g., schedule execution of concurrent processes)

Hybrid Systems Simulation



Hybrid System Simulation

A simulator for hybrid systems must capture different types of behaviors:

- Continuos Time
- Discrete Events
- FSMs …

and resolve the domain interface problems.

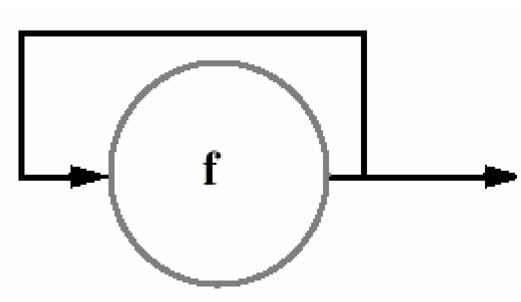
Continuous Time

- **Model of computation is DISCRETE TIME**
  - **All variables are computed at each time point**
    - **no run-time scheduling decisions on variable computation**
  - **Time interval can be**
    - **fixed (bad for stiff systems), but no run-time decision**
    - **variable (sophisticated solvers have this)**
      - Variable time step algorithm predicts a time step that will satisfy accuracy criterion based on previous behavior
      - After actual computation, step may be rejected because constraints are violated
      - Run-time scheduling

Discrete Domain

- Two basic techniques:
  - Zero-time assumption:
    - Static scheduling of computation
    - Can be done off-line for maximum efficiency (cycle-based simulation)
  - Components modeled with delay (Discrete Event Model).
    - All components evaluated at the same time-point always (wasteful)
    - Follow reaction to events: schedule components whose inputs have changed (assumes internal dynamics completely captured by pure delay) Selective-trace event-driven simulation.

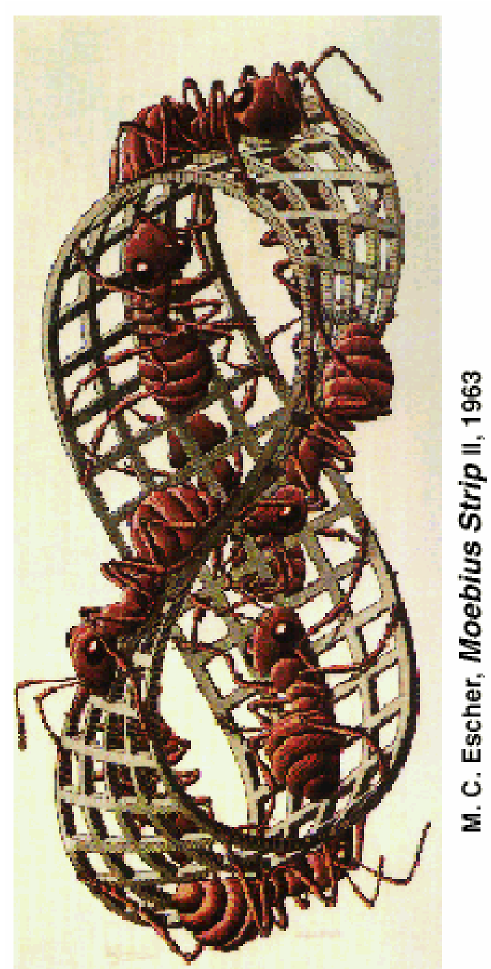
Zero-time Loops



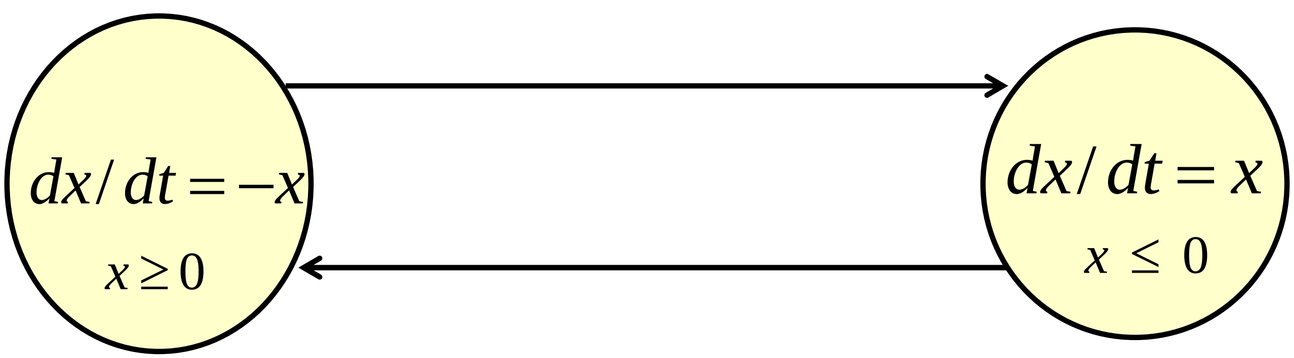
For $f : S \to S$, define the ***semantics*** to be a ***fixed point*** of f

i. e. s such that

$$f(\mathbf{s}) = \mathbf{s}$$



M. C. Escher, *Moebius Strip* II, 1963



Synchronization Problem

- "Synchronization" between domains:
  - sample the continuous time interface variables
  - integrate discrete event interface signals
  - detect guards and invariants (zero crossing detection)

Simulator Architecture

- One simulator (e.g. Ptolemy)
  - different algorithms for each domain and unique scheduler
- N simulators (e.g. Simulink-StateFlow, Simulink-Bones, Simulink-VCC)
  - One simulator per domain (different schedulers per domain) and communication among simulators.
  - Scheduler works by transferring control to simulator
  - Much less efficient but easier to do!

Invariant Detection

- **An approach:**
  - the discrete event simulator checks the conditions sampling the continuos time variables
- **Advantages:**
  - easiest implementation
  - strong separation between the two domains
- **Drawbacks:**
  - high precision detection reached only with long simulation time.
  - high inter-process communication overhead
- **Partial Solution:**
  - Simulation look-ahead

Outline

- Introduction to WP
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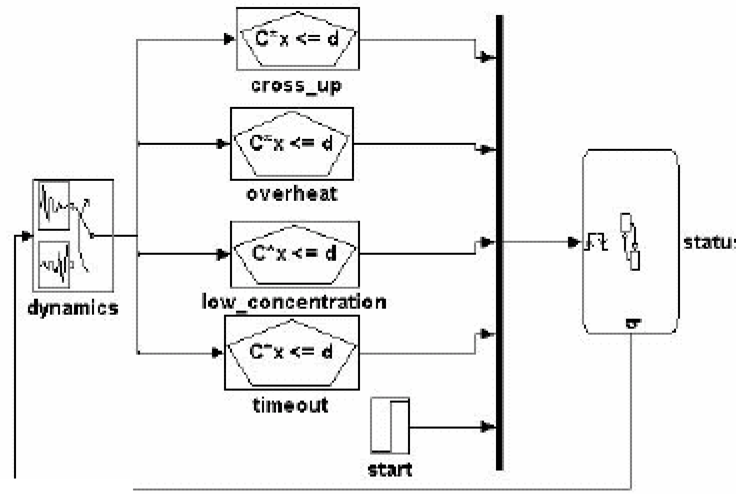
CheckMate

Simulink/Stateflow Front End (graphical editing, simulation)

Threshold-event-driven Hybrid Systems (TEDHS)



Source: B. Krogh

The CheckMate Model: TEDHS

Three parts:

- Switched Continuous System (SCS), that takes in the discrete-valued input u and produces continuous state vector x as output into TEG.
- Threshold Event Generator (TEG), produces an event when a component of x crosses a corresponding threshold from the specified direction (rising, falling, or both) and feeds FSM.
- Finite State Machine (FSM), whose output, in turn, drives the continuous dynamics of the SCS.

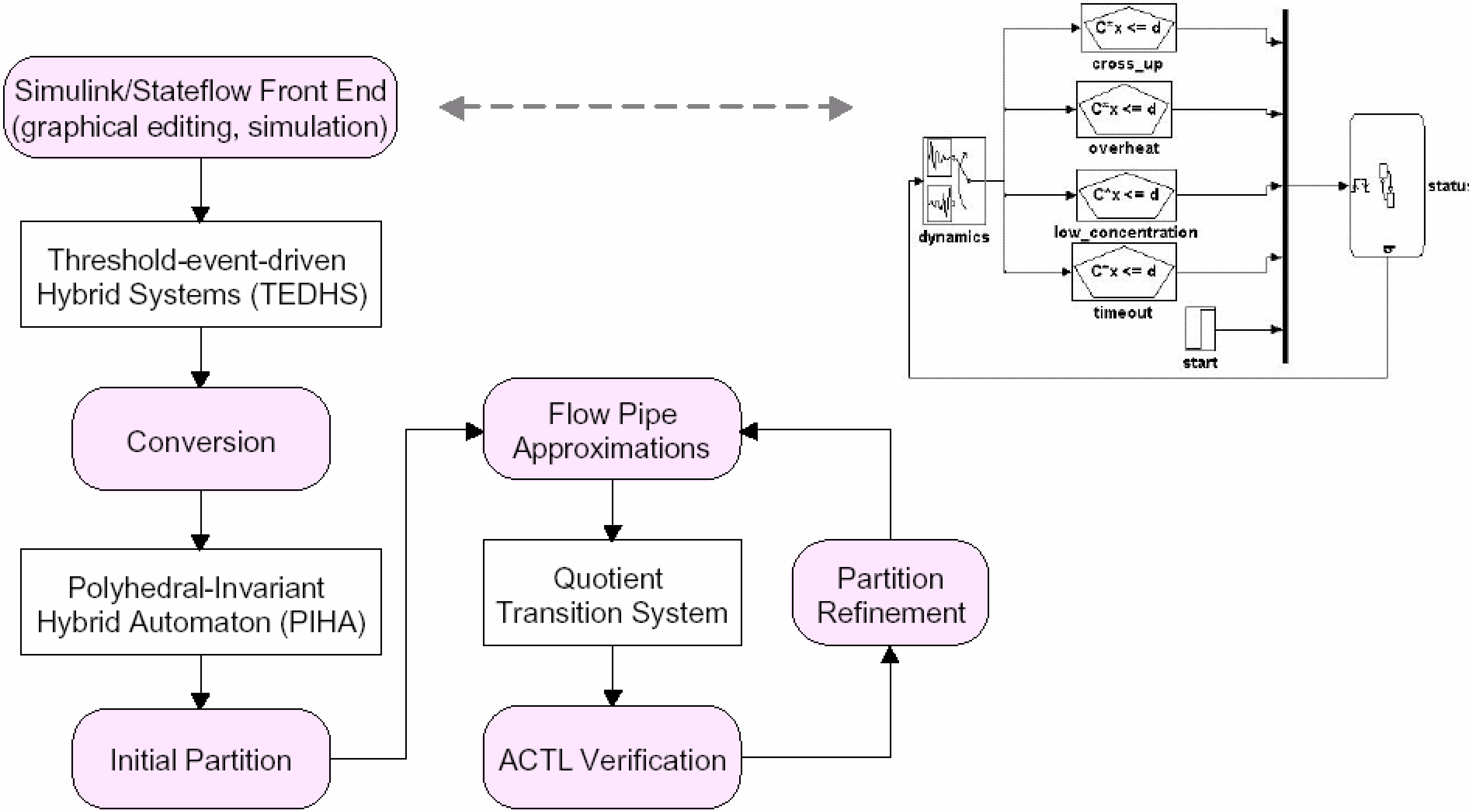
Deriving DES Models from Hybrid System Models

Given a hybrid system:

1. Consider behavior *only* at event times
2. Compute reachability between *sets* of continuous states
3. Perform analysis/control synthesis using the resulting transition system
4. If necessary, refine sets of states & return to 2.

Source: B. Krogh

CheckMate



Source: B. Krogh

The Polyhedral Invariant Hybrid Automaton

A PIHA is a hybrid automaton with the following restrictions:

- The continuous dynamics for each location is governed by an ordinary differential equation (ODE).
- Each guard condition is a linear inequality (**a hyper-plane guard**).
- Each reset condition is an **identity**.
- For the hybrid automaton to remain in any location, of the hybrid system **all guard conditions must be false**. This restriction implies that the invariant condition for any location is the convex polyhedron defined by conjunction of the complements of the guards. This gives rise to the name **polyhedral-invariant hybrid automaton.**

CheckMate Summary

- Integrated with Matlab/Simulink/StateFlow
- Limited semantics to simplify analysis and allow formal verification
- Uses Simulink constructs to enter data
- Based on reachability analysis to abstract continuous away
- Can perform simulation, partial and complete verification
- Computationally complex…

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What is Charon?

Charon is a high-level modeling language and a design environment for hybrid systems reflecting the current state of the art both in formal and object oriented methods (UML).

- Architectural Hierarchy (Agents)
- Behavioral Hierarchy (Modes)

- Charon toolkit
  - Syntax-directed editor
  - Parser and type checker
  - Global simulator
  - Plotter (from Ptolemy)

Language Summary

- Individual components described as agents
- Individual behaviors described as modes
- Support for concurrency
  - Shared variables as well as message passing
- Support for discrete and continuous behavior
- Well-defined formal semantics

Continuous Behavior in Charon

- Differential Constraints
  - write Position robot_Pos;
  - diff diffStop {d(robot_ Pos.x)=0.0; d(robot_ Pos.y)=1.0;}

- Algebraic Equations
  - write real robotEST;
  - read x ;
  - alge contEST { robotEST = foo(x) + bar(x); }

- Invariant Constraints in Modes
  - inv invTUCost { lub <= x <= gub; }

Simulation in Charon

- In the present approach, a program-specific simulator is *generated* from the Charon program
- Each object of the Charon program is converted into an executable Java object
- Together with a program-independent core, these objects implement behavior of the program (Compiled-Code simulator)

Future Extensions

- Graphical input language
- Modular simulation
- Model Checker

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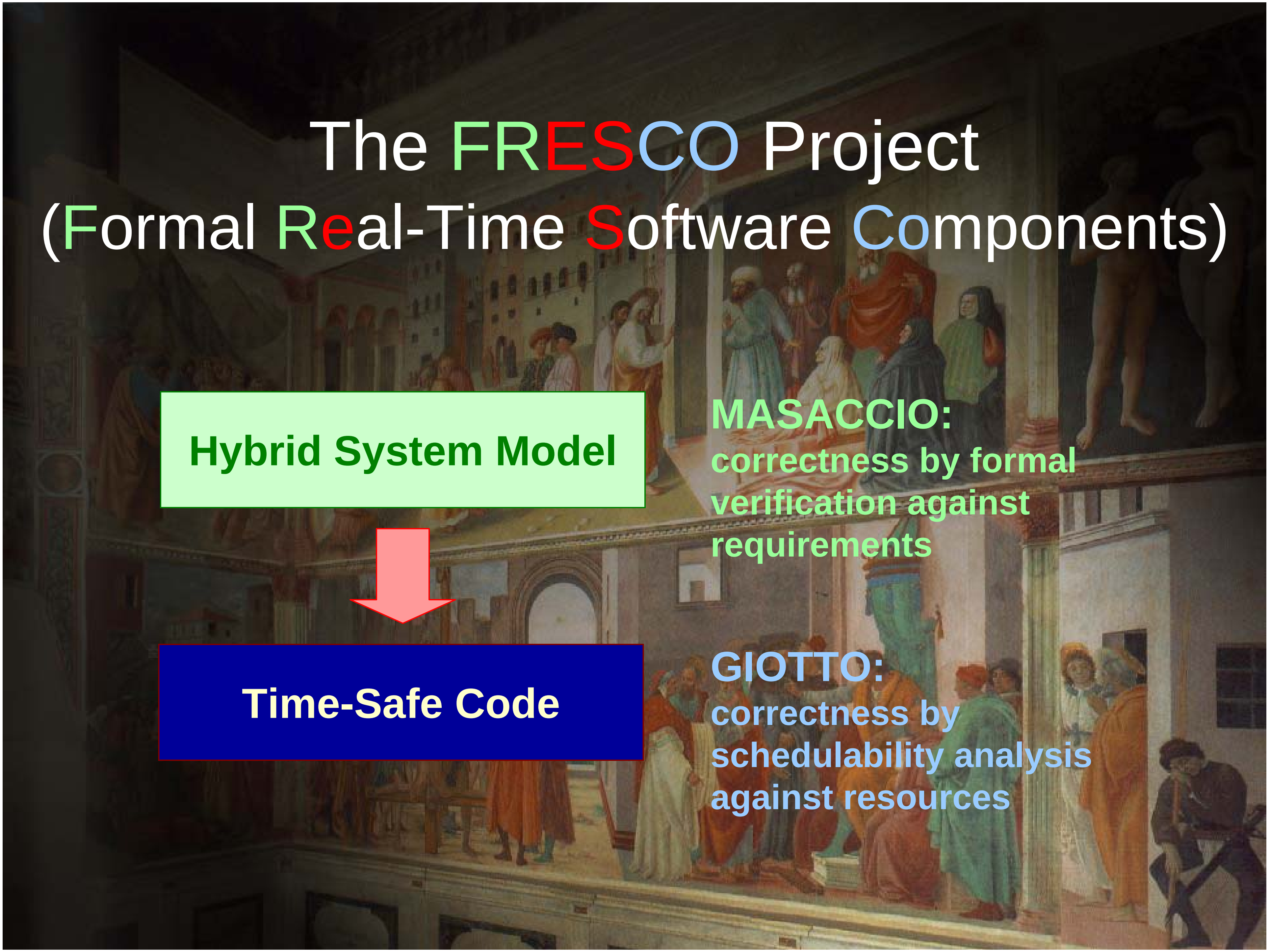
The FRESCO Project
(Formal Real-Time Software Components)



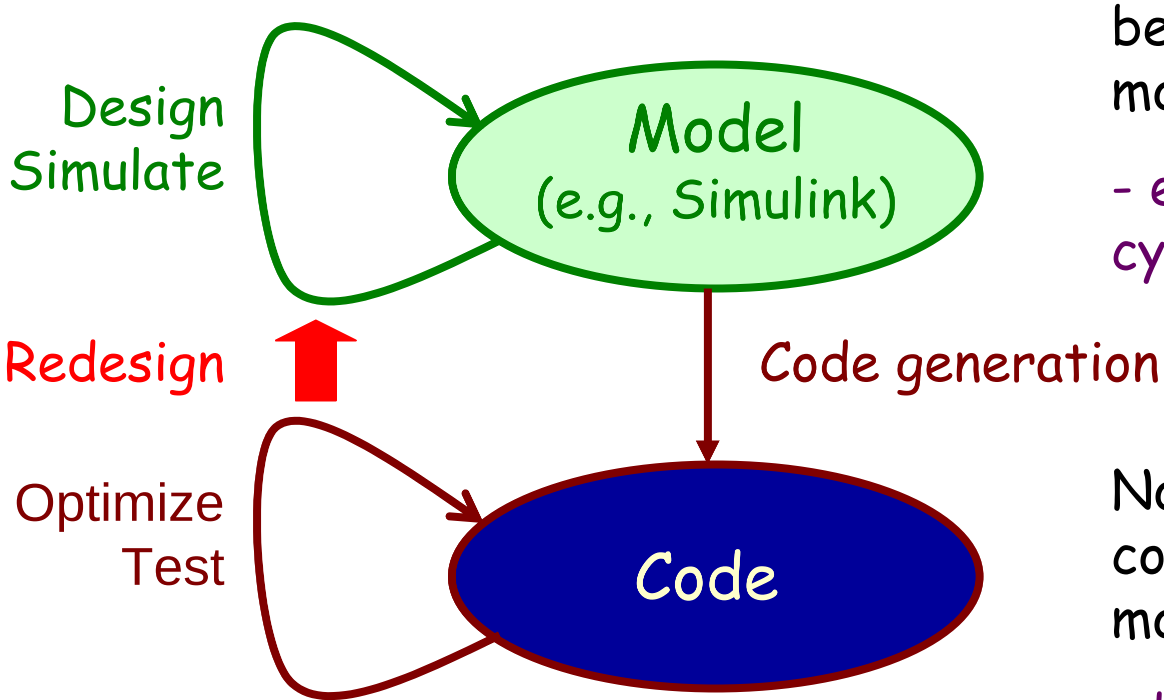
Hybrid System Model

Time-Safe Code

MASACCIO:
correctness by formal verification against requirements

GIOTTO:
correctness by schedulability analysis against resources

Embedded Software Design: Current State



No formal connection between requirements, model, and resources:

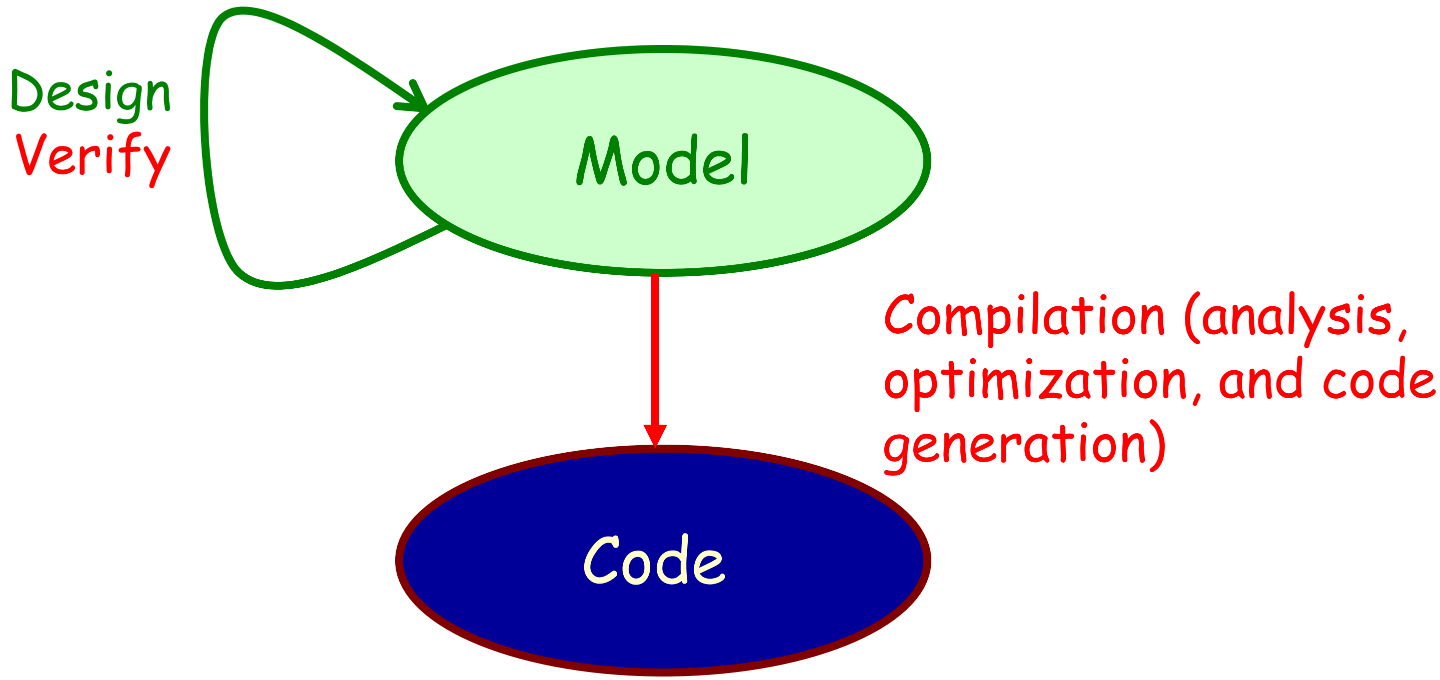
- expensive development cycle iterates all stages

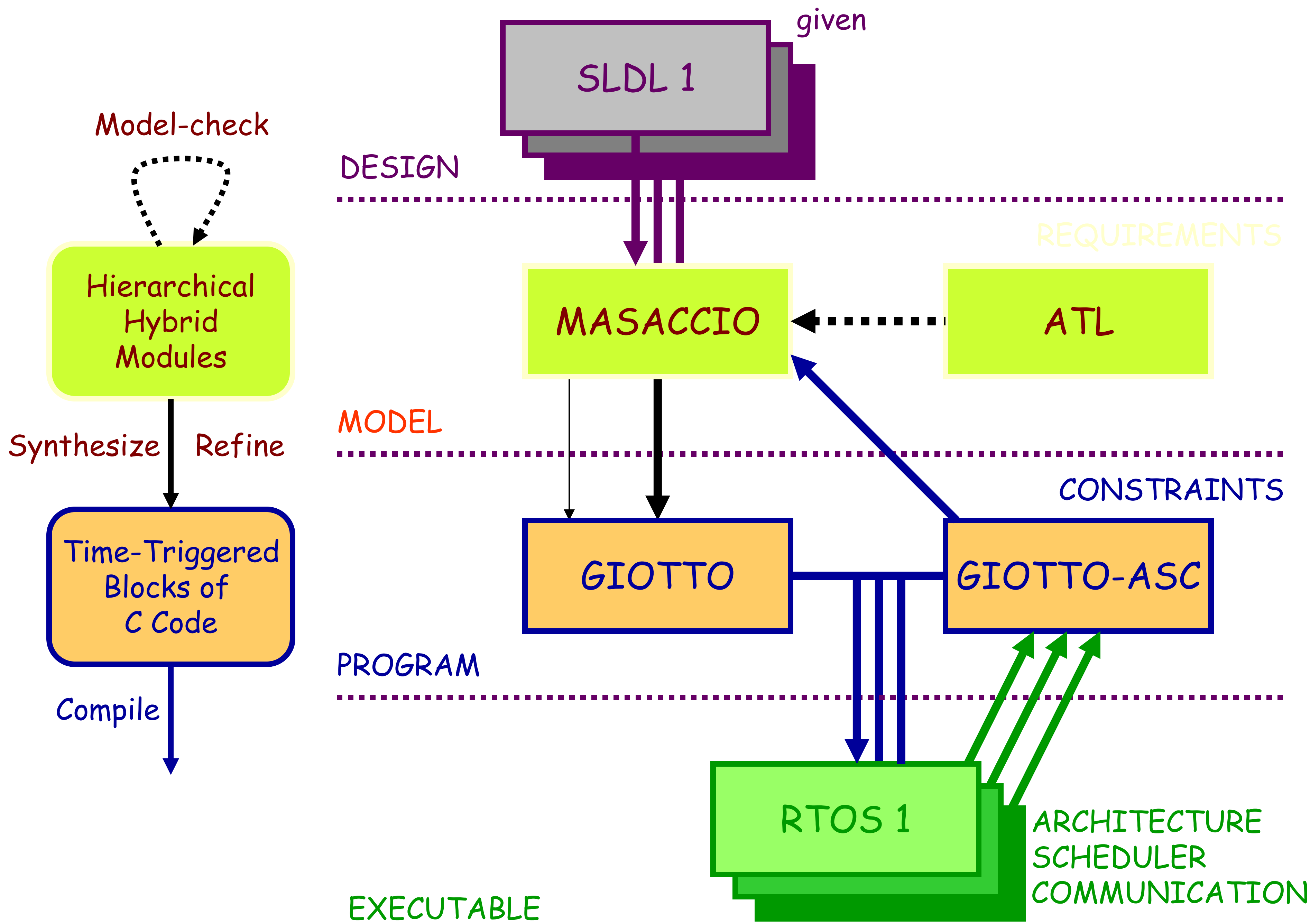
No exact correspondence between model and code:

-difficult to upgrade code

-difficult to reuse code

Embedded Software Design: UCB and PARADES Vision





given
SLDL 1
Model-check
DESIGN
REQUIREMENTS
Hierarchical
Hybrid
Modules
MASACCIO
ATL
MODEL
Synthesize
Refine
CONSTRAINTS
Time-Triggered
Blocks of
C Code
GIOTTO
GIOTTO-ASC
PROGRAM
Compile
RTOS 1
ARCHITECTURE
SCHEDULER
COMMUNICATION
EXECUTABLE

MASACCIO

Semantics:

Component = interface + behaviors

Interface (the "statics"):

- Variables: input/output, discrete/continuous (data)
- Locations: entry/exit (control)

Behavior (the "dynamics"):

- Jumps: all variables may change (instantaneous)
- Flows: continuous variables evolve (real-valued duration)

Masaccio & Charon: an informal comparison

Charon's hierarchy:

- architectural -> agents -> parallel composition
- behavioral -> modes -> parallel & serial comp

Masaccio's hierarchy:

- both architectural & behavioral -> components -> parallel & serial comp.

Features:

- Charon -> Simulation; more developed
- Masaccio -> Formal Verification; few papers and few applications; focusing on Giotto at the moment